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CHAPTER

8 Systems of Linear Equations

A keypunch operator at a local firm works for \$9 per hour while an entry-level typist works for \$6.50 per hour. The total pay for an 8-hour day is \$476. If there are two more typists than keypunch operators, how many keypunch operators does the firm employ?

8–1 ■ Solutions of systems of linear equations by graphing

In chapter 7, we showed by graphs the relationship between the variables in a linear equation in two variables. We now consider the relationship that exists when *two* linear equations in two variables are graphed on the same coordinate plane. Two or more linear equations involving the same variables form a **system of linear equations.** Here are examples of systems of linear equations.

I	II	III	IV
$4x - y = 1$	$3x - 7y = 5$	$x = 3y + 1$	$x = 4$
$x + y = 4$	$5x + 2y = 11$	$x + 3y = 4$	$3x - 2y = 5$

To solve any one of these systems of linear equations, we must find the ordered pair(s) of real numbers (x,y) that satisfy both equations at the same time.

Solutions of systems of linear equations

A **solution** of a system of linear equations is an ordered pair of numbers that satisfies (makes true) all equations in the system.

■ Example 8–1 A

Determine whether the given ordered pair is a solution of the system of linear equations.

1. $4x - y = 1$
$x + y = 4$ $\quad (1,3)$

$$\begin{aligned} 4x - y &= 1 \\ 4(1) - (3) &= 1 \\ 4 - 3 &= 1 \\ 1 &= 1 \end{aligned}$$

Replace x with 1 and y with 3

(True)

$$\begin{aligned} x + y &= 4 \\ (1) + (3) &= 4 \\ 4 &= 4 \end{aligned}$$

(True)

Both resulting statements are true. Therefore, the ordered pair (1,3) satisfies both equations and is a solution.

2. $5x + 3y = -20$
$7x + 2y = -17$ $\quad (-4,0)$

$$\begin{aligned} 5x + 3y &= -20 \\ 5(-4) + 3(0) &= -20 \\ -20 + 0 &= -20 \\ -20 &= -20 \end{aligned}$$

Replace x with -4 and y with 0

(True)

$$\begin{aligned} 7x + 2y &= -17 \\ 7(-4) + 2(0) &= -17 \\ -28 + 0 &= -17 \\ -28 &= -17 \end{aligned}$$

(False)

The ordered pair $(-4,0)$ is *not* a solution of the system since it does not satisfy the equation $7x + 2y = -17$.

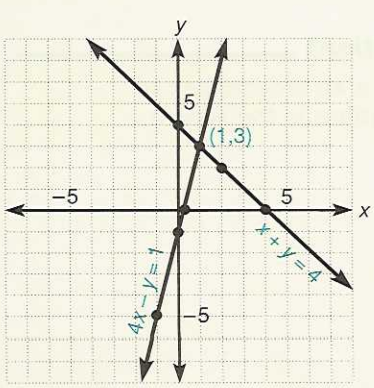
▶ ***Quick check*** Determine if the ordered pair (1,1) is a solution of the system.
$3x - y = 2$
$4x + 3y = 9$ ■

Graphical solutions

The first method that we shall examine is finding the solution of a system of linear equations by **graphing.** We are looking for the coordinates of any point where the graphs intersect. This point of intersection will give a solution of the system. Using the methods for graphing that we learned in chapter 7, we graph each equation on the same coordinate plane. Using the x- and y-intercepts and a checkpoint, we graph each equation.

■ Example 8–1 B

Find the solution of each system by graphing.



1. $4x - y = 1$
$x + y = 4$

$4x - y = 1$

x	y	
0	-1	y-intercept; $x = 0$
$\frac{1}{4}$	0	x-intercept; $y = 0$
-1	-5	Checkpoint

$x + y = 4$

x	y
0	4
4	0
2	2

It appears the point of intersection of the two graphs is the point (1,3). We must check this as the possible solution.

$$\begin{aligned} 4x - y &= 1 \\ 4(1) - (3) &= 1 \quad \text{Replace } x \text{ with 1 and } y \text{ with 3} \\ 4 - 3 &= 1 \\ 1 &= 1 \quad \text{(True)} \end{aligned}$$

$$\begin{aligned} x + y &= 4 \\ (1) + (3) &= 4 \\ 4 &= 4 \quad \text{(True)} \end{aligned}$$

The solution of the system is the ordered pair (1,3).

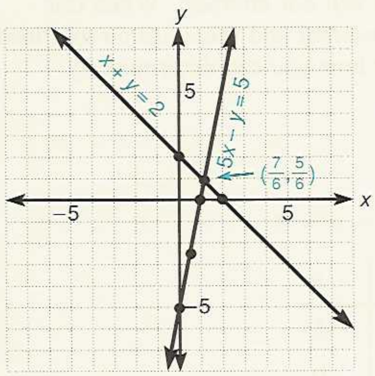
2. $5x - y = 5$
$x + y = 2$

$5x - y = 5$

x	y	
0	-5	y-intercept; $x = 0$
1	0	x-intercept; $y = 0$
$\frac{1}{2}$	$-\frac{5}{2}$	Checkpoint

$x + y = 2$

x	y
0	2
2	0
1	1



An estimate of the point of intersection is (1,1). A check shows this is not a correct solution.

$$\begin{aligned} 5x - y &= 5 \\ 5(1) - (1) &= 5 \quad \text{Replace } x \text{ with 1 and } y \text{ with 1} \\ 4 &= 5 \quad \text{(False)} \end{aligned}$$

$$\begin{aligned} x + y &= 2 \\ (1) + (1) &= 2 \\ 2 &= 2 \quad \text{(True)} \end{aligned}$$

We will see in section 8–2 that the actual point of intersection is $\left(\frac{7}{6},\frac{5}{6}\right)$.

▶ ***Quick check*** Estimate the solution of the system by graphing.
$3x - y = 6$
$2y + x = 4$ ■

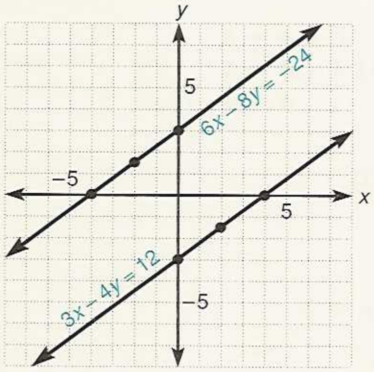
It should be obvious that our graphical method of finding the solution may not be exact because the drawing of the graph depends on measuring devices. Inaccuracies are thereby often produced in the graphing process.

When the graph of a system consists of intersecting lines and thus has a simultaneous solution, the system is said to be **consistent** and **independent.**

Inconsistent and dependent systems

Sometimes the graphs of the two equations in the system do not intersect at all (are parallel lines) or are one and the same line.

■ Example 8–1 C



1. $3x - 4y = 12$
$6x - 8y = -24$

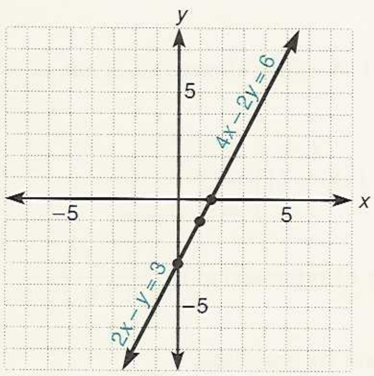
$3x - 4y = 12$

x	y	
0	-3	y-intercept; $x = 0$
4	0	x-intercept; $y = 0$
2	$-\frac{3}{2}$	Checkpoint

$6x - 8y = -24$

x	y
0	3
-4	0
-2	$\frac{3}{2}$

The lines appear to be parallel (they are) and will not intersect. When this condition exists, the system is said to be *inconsistent* and there are no solutions (no points of intersection). We will show later how you can determine by algebra if the lines are or are not parallel.



2. $4x - 2y = 6$
$2x - y = 3$

$4x - 2y = 6$

x	y	
0	-3	y-intercept; $x = 0$
$\frac{3}{2}$	0	x-intercept; $y = 0$
1	-1	Checkpoint

$2x - y = 3$

x	y
0	-3
$\frac{3}{2}$	0
1	-1

We observe that the two intercepts and checkpoint $(1, -1)$, are the same for both lines. Therefore, the two lines coincide and the graph is a single line. Such a system is called a **dependent** system of equations. In a dependent system of equations, an unlimited number of solutions are possible because each ordered pair that satisfies the first equation will also satisfy the second. ■

To solve a system of linear equations by graphing

1. Graph each equation on the same set of axes.
2. Their graphs may intersect in
   a. one point—one solution (the system is *consistent* and *independent*).
   b. no points—the lines are parallel and there are no solutions (the system is *inconsistent*).
   c. all points—the lines are one and the same (the system is *dependent*).
3. If the graphs intersect in one point, check the coordinates of the point in each equation.

Mastery points

Can you

- Determine if an ordered pair is a solution of a system of linear equations?
- Solve a system of linear equations by graphing?
- Recognize an inconsistent and a dependent system of linear equations?

Exercise 8–1

Determine if the given ordered pair is a solution to the system of equations. See example 8–1 A.

Example Determine if the ordered pair (1,1) is a simultaneous solution of the system. $3x - y = 2$
$4x + 3y = 9$

Solution Substitute 1 for x and 1 for y in each equation.

$3(1) - (1) = 2$ Replace x with 1 and y with 1 $\quad 4(1) + 3(1) = 9$
$3 - 1 = 2 \quad\quad 4 + 3 = 9$
$2 = 2$ (True) $\quad\quad 7 = 9$ (False)

Since (1,1) satisfies only one equation, it is *not* a solution of the system of equations.

1. $x + y = 2$, $3x - y = 10$ $\quad(3,-1)$
2. $x + 2y = 4$, $x + 4y = 10$ $\quad(-2,3)$
3. $x - y = 1$, $x + y = 5$ $\quad(3,2)$
4. $3x - y = -10$, $3x + y = 4$ $\quad(-1,7)$
5. $2x + y = 2$, $6x - y = 22$ $\quad(3,-4)$
6. $x - 3y = -1$, $3x + y = -2$ $\quad(-1,0)$
7. $3x + 4y = 14$, $2x + y = 6$ $\quad(1,4)$
8. $3x + y = 12$, $2x + 3y = 15$ $\quad(2,6)$
9. $x = 6$, $2x - y = 3$ $\quad(6,9)$
10. $y = 4x - 3$, $y = -1$ $\quad(-1,-1)$
11. $3x - 2y = 4$, $y = x + 3$ $\quad(10,2)$
12. $y = 2 - 5x$, $x - y = 6$ $\quad(-1,5)$

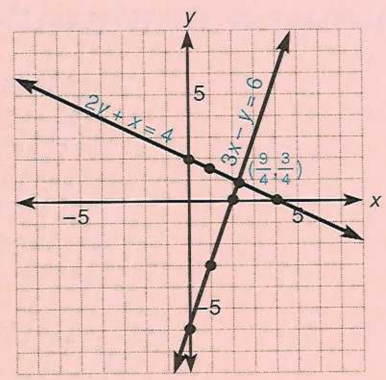
Estimate the solution of each system by graphing. All answers in the back of the book will be given exactly. If the system is inconsistent or dependent, so state. See examples 8–1 B, C.

Example $3x - y = 6 \quad\quad 2y + x = 4$

Solution $3x - y = 6 \quad\quad 2y + x = 4$

x	y	
0	-6	y-intercept
2	0	x-intercept
1	-3	Checkpoint

x	y
0	2
4	0
1	$\frac{3}{2}$



We estimate the solution to be $\left(\frac{9}{4},\frac{3}{4}\right)$.

Note The actual solution is $\left(\frac{16}{7},\frac{6}{7}\right)$.

13. $x - y = 1$
$x + y = 5$

14. $x + 2y = 4$
$x + 4y = 10$

15. $3x - y = 10$
$x + y = 2$

16. $x - y = 3$
$2x + 3y = 11$

17. $2x - y = 3$
$y + 2x = -7$

18. $y = 1$
$3x - 2y = -9$

19. $2x + 3y = 13$
$3x - 2y = 0$

20. $3x + y = 10$
$6x + 2y = 5$

21. $2x + y = 2$
$-6x - 3y = 6$

22. $x = 4$
$3x + 2y = 4$

23. $3x + y = -3$
$x = -1$

24. $y + 2x = 6$
$4y + 3x = 4$

25. $3x + y = 3$
$6x + 2y = 3$

26. $2x - y = 3$
$6x - 3y = 9$

27. $5x + y = -5$
$-10x - 2y = 10$

28. $\frac{3}{2}x - \frac{1}{2}y = -5$
$x + \frac{1}{3}y = \frac{4}{3}$

29. $\frac{1}{3}x + \frac{1}{6}y = 1$
$\frac{1}{4}x + \frac{1}{3}y = 1$

30. $6x - 5y = 14$
$y = 8$

Review exercises

1. Find the solution set of the equation $\frac{3x}{2} - \frac{x}{4} = 1$. See section 6–5.

2. Divide as indicated. $(3x^3 - x^2 + 1) \div (x + 3)$. See section 5–3.

Find the solution of the following inequalities. See section 2–6.

3. $4y - 3 \leq 2y + 7$

4. $-5 < 2x + 1 \leq 7$

Perform the indicated operations. See sections 3–2 and 3–3.

5. $x^{-3} \cdot x^5 \cdot x^2$

6. $x^3(x^2 - 2x + 1)$

7. $(2x - y)^2$

8–2 ■ Solutions of systems of linear equations by elimination

In section 8–1, we found the solution of a system of consistent and independent linear equations by graphing the equations and by observing the coordinates of the point of intersection of the two lines. This technique will often produce only an approximate solution.

An algebraic method for obtaining an *exact* solution to a system involves the operation of *addition*, together with the following property of real numbers.

Addition property of equality

Given a, b, c, and d are real numbers, if $a = b$ and $c = d$,

$$a + c = b + d$$

Concept

Adding equal quantities to each member of an equation will result in equal sums.

This property of real numbers is used to *eliminate* one of the variables in the system. The resulting single equation has only one unknown that we solve to obtain one of the components of the solution. We illustrate this property's use in the following examples.

■ Example 8–2 A

Find the solution set of the following systems of linear equations by elimination.

1. $2x - y = 4$
$x + y = 2$

Adding the left members and the right members, we obtain

$$\begin{array}{rll} 2x - y &= 4 & \\ \underline{x + y} &\underline{= 2} & \\ 3x \quad\; &= 6 & \text{Add left and right members} \\ x &= 2 & \text{Divide each member by 3} \end{array}$$

We have found the value of x to be 2. To find y, we replace x with 2 in *either* of the equations and solve for y.

$$\begin{array}{rll} x + y &= 2 & \text{Choose either equation } (x + y = 2) \\ (2) + y &= 2 & \text{Replace } x \text{ with 2} \\ y &= 0 & \text{Subtract 2 from each member} \end{array}$$

The solution of our system is (2,0). The solution set is $\{(2,0)\}$.

Check:

$$\begin{array}{rl} 2x - y &= 4 \\ 2(2) - (0) &= 4 \\ 4 - 0 &= 4 \\ 4 &= 4 \quad \text{(True)} \end{array} \qquad \begin{array}{rl} x + y &= 2 \\ (2) + (0) &= 2 \\ 2 &= 2 \quad \text{(True)} \end{array}$$

2. $x = 4 - 3y$
$2x - 3y = 2$

To solve the system by elimination, like terms must be in a column and the equations must be in standard form.

$$\begin{array}{rll} x + 3y &= 4 & \text{Add } 3y \text{ to each member of first equation} \\ \underline{2x - 3y} &\underline{= 2} & \\ 3x \quad\;\; &= 6 & \text{Add left and right members} \\ x &= 2 & \text{Divide each member by 3} \end{array}$$

Replace x with 2 in either equation and solve for y.

$$\begin{array}{rll} x + 3y &= 4 & \text{Choose either equation } (x + 3y = 4) \\ (2) + 3y &= 4 & \text{Replace } x \text{ with 2} \\ 3y &= 2 & \text{Subtract 2 from each member} \\ y &= \dfrac{2}{3} & \text{Divide each member by 3} \end{array}$$

The solution of the system is the ordered pair $\left(2,\frac{2}{3}\right)$. The solution set is $\left\{\left(2,\frac{2}{3}\right)\right\}$. Check the solution in *both original* equations. ■

In the previous examples, we observe that one of the variables was easily eliminated by addition because in the original equations that variable had coefficients that were opposites. When this condition is not present in the equation, we must first use the following procedure to obtain the necessary opposite coefficients.

> Multiply one or both equations by some nonzero constant so that equivalent equations are formed where one of the variables has coefficients that are opposites.

Example 8–2 B

Find the solution set of each system of linear equations by elimination.

1. $3x + 4y = 14$ | | $3x + 4y = 14$
$2x + y = 6$ | Multiply each member by -4 → | $-8x - 4y = -24$
| Add the equations | $-5x = -10$
| Solve for x | $x = 2$

$3(2) + 4y = 14$ Replace x with 2 in $3x + 4y = 14$
$6 + 4y = 14$ Solve for y
$4y = 8$
$y = 2$

Solution: (2,2). The solution set is $\{(2,2)\}$. Check the solution.

2. $7x + 5y = -9$ | Multiply each member by 2 → | $14x + 10y = -18$
$3x - 2y = -8$ | Multiply each member by 5 → | $15x - 10y = -40$
| Add the equations | $29x = -58$
| Solve for x | $x = -2$

Using $7x + 5y = -9$, we substitute and solve for y.

$7(-2) + 5y = -9$ Replace x with -2 in $7x + 5y = -9$
$-14 + 5y = -9$ Solve for y
$5y = 5$
$y = 1$

Solution: $(-2,1)$. The solution set is $\{(-2,1)\}$. Check the solution.

Note In the previous example, both equations had to be multiplied by a constant to form opposite coefficients of y. We could have eliminated x by multiplying the first equation by 3 and the second equation by -7, forming opposite coefficients of 21 and -21. It is an arbitrary choice to decide which variable to eliminate, but for ease in solving the system, it is best to eliminate the variable whose opposite coefficients will be least in value.

3. $\frac{1}{2}x + \frac{2}{3}y = \frac{1}{6}$
$\frac{3}{4}x - \frac{1}{4}y = \frac{1}{2}$

The first step in this problem will be to clear the fractions.

$\frac{1}{2}x + \frac{2}{3}y = \frac{1}{6}$ Multiply each member by the LCD, 6 $6\left(\frac{1}{2}x + \frac{2}{3}y\right) = 6 \cdot \frac{1}{6}$

$3x + 4y = 1$

$\frac{3}{4}x - \frac{1}{4}y = \frac{1}{2}$ Multiply each member by the LCD, 4 $4\left(\frac{3}{4}x - \frac{1}{4}y\right) = 4 \cdot \frac{1}{2}$

$3x - y = 2$

We now solve the system.

$3x + 4y = 1$

$3x - y = 2$ Multiply each member by $(-1) \rightarrow$

$$\begin{aligned} 3x + 4y &= 1 \\ -3x + y &= -2 \\ \hline 5y &= -1 \\ y &= -\frac{1}{5} \end{aligned}$$

Add the equations

Solve for y

$3x - \left(-\frac{1}{5}\right) = 2$ Replace y with $-\frac{1}{5}$ in $3x - y = 2$

$3x + \frac{1}{5} = 2$ Definition of subtraction

$15x + 1 = 10$ Multiply each term by the LCD, 5

$15x = 9$ Subtract 1 from each member

$x = \frac{3}{5}$ Divide each term by 15 and reduce

Solution: $\left(\frac{3}{5}, -\frac{1}{5}\right)$. The solution set is $\left\{\left(\frac{3}{5}, -\frac{1}{5}\right)\right\}$.

4. $4x - 2y = 6$

$2x - y = 3$ Multiply each member by $(-2) \rightarrow$

Add the equations

$$\begin{aligned} 4x - 2y &= 6 \\ -4x + 2y &= -6 \\ \hline 0x + 0y &= 0 \\ 0 &= 0 \quad \text{(True)} \end{aligned}$$

At this point, both variables have been eliminated, and we have formed the true statement $0 = 0$. This indicates that any ordered pair that satisfies either equation is a solution and there are an unlimited number of solutions. The system is *dependent* and the solution set is all ordered pairs satisfying equation $2x - y = 3$ or $4x - 2y = 6$. The same line is graphed.

5. $3x - 4y = 12$ Multiply each member by $(-2) \rightarrow$

$6x - 8y = -24$

Add the equations

$$\begin{aligned} -6x + 8y &= -24 \\ 6x - 8y &= -24 \\ \hline 0x + 0y &= -48 \\ 0 &= -48 \quad \text{(False)} \end{aligned}$$

The resulting statement, $0 = -48$, is false, which indicates there is no ordered pair that can satisfy this system. The system is *inconsistent* and the solution set is $\emptyset$ (the null set). Parallel lines are graphed.

▶ ***Quick check*** Find the solution set of the system of equations by elimination.

$5x - 2y = 0$

$y - 3x = 4$

To solve a system of linear equations by elimination

1. If needed multiply the members of one or both equations by a constant that will make the coefficients of one of the variables opposites of each other.
2. Add the corresponding members of the equations.
3. Solve this new equation for the remaining variable.
4. Substitute the value found in step 3 into either of the original equations and solve this equation for the other variable. The solution is the ordered pair of numbers obtained in step 3 and this step.
5. If, when adding in step 2,
   a. both variables are eliminated and a *false statement* is obtained, there are *no solutions* and the system is *inconsistent* (parallel lines).
   b. both variables are eliminated and a *true statement* such as $0 = 0$ is obtained, there are infinitely *many* solutions and the system is *dependent* (same line).
6. If the system has a solution found in step 4, check the solution in both equations.

Mastery points

Can you

- Solve a system of linear equations by elimination?
- Recognize a dependent or an inconsistent system of linear equations while solving by elimination?

Exercise 8–2

Find the solution set of the following systems of linear equations by elimination. If a system is inconsistent or dependent, so state. See examples 8–2 A and B.

Example $5x - 2y = 0$
$y - 3x = 4$

Solution Rewrite the second equation to get the system.

$$\begin{aligned} 5x - 2y &= 0 \\ -3x + y &= 4 \end{aligned}$$

Multiply each term by 2 →

$$\begin{aligned} 5x - 2y &= 0 \\ -6x + 2y &= 8 \end{aligned}$$

Add equations $\quad -x = 8$

Multiply by -1 $\quad x = -8$

$5(-8) - 2y = 0$ Replace x with -8 in $5x - 2y = 0$

$-40 - 2y = 0$ Solve for y

$-2y = 40$

$y = -20$

Solution: $(-8,-20)$. The solution set is $\{(-8,-20)\}$.

1. $x - y = 1$
$x + y = 5$

2. $x + y = 2$
$3x - y = 10$

3. $3x - y = 10$
$6x + y = 5$

4. $2x + y = 2$
$-6x - y = 4$

5. $x - 2y = 4$
$-x + 2y = 4$

6. $-x + y = 6$
$x + 4y = 4$

7. $-3x + y = -3$
$3x - 3y = -1$

8. $2x + y = 1$
$-2x + y = 1$

9. $x + 2y = 4$
$x + 4y = 10$

$\boxed{10.}$ $x - y = 3$
$2x + 3y = 11$

11. $3x - 2y = -9$
$2x + y = 1$

12. $5x - y = 4$
$x + 3y = 2$

13. $4x + y = 11$
$2x + 3y = 4$

14. $x - 2y = 1$
$3x - 6y = 3$

15. $8x - 4y = 12$
$2x - y = 3$

16. $-2x + 3y = 6$
$4x + y = 1$

17. $10x + y = 5$
$2x + 5y = -23$

18. $10x - 5y = 7$
$2x - y = 4$

$\boxed{19.}$ $-6x + 3y = 9$
$2x - y = -3$

20. $3x - 2y = 6$
$4x - 3y = 7$

21. $5x + 2y = 3$
$3x - 3y = 4$

22. $4x + 7y = 11$
$8x - 3y = -4$

23. $5x - 2y = 4$
$3x + 3y = 5$

24. $5x - 3y = 34$
$4x + 5y = 5$

25. $3x - 2y = 19$
$5x + 3y = 19$

26. $4x + 5y = -2$
$3x - 2y = -36$

$\boxed{27.}$ $\frac{1}{2}x + \frac{1}{3}y = 1$
$\frac{2}{3}x - \frac{1}{4}y = \frac{1}{12}$

28. $\frac{2}{3}x - \frac{1}{4}y = 4$
$\frac{1}{3}x + y = 2$

29. $\frac{1}{2}x + \frac{2}{5}y = \frac{7}{10}$
$\frac{3}{2}x + \frac{6}{5}y = \frac{3}{10}$

30. $\frac{6}{7}x - \frac{3}{5}y = \frac{9}{10}$
$\frac{2}{7}x - \frac{1}{5}y = \frac{5}{10}$

31. $\frac{9}{2}x + \frac{1}{2}y = -10$
$\frac{1}{3}x + \frac{1}{4}y = \frac{3}{4}$

32. $\frac{7}{2}x - y = -\frac{17}{2}$
$\frac{2}{3}x + \frac{1}{3}y = 1$

33. $(0.4)x - (0.7)y = 0.7$
$(0.6)x - (0.5)y = 2.7$

$\boxed{34.}$ $x + (0.4)y = 3.4$
$(0.6)x - (1.4)y = 0.4$

$\boxed{35.}$ Let x represent the width of a rectangle and y represent the length of the rectangle. Write an equation stating that the length is five times the width.

36. The perimeter of the rectangle in exercise 35 is 60 inches. If the perimeter of a rectangle equals twice the width plus twice the length, write an equation for the perimeter of the rectangle.

37. Let x represent the speed of automobile A and y represent the speed of automobile B. Write an equation stating that automobile B travels 20 mph faster than automobile A.

$\boxed{38.}$ If the two automobiles in exercise 37 travel in opposite directions, write an equation that states the two automobiles are 500 miles apart after 3 hours. (*Note:* distance = speed times the time traveled)

39. Let x represent the amount of money Jane invests at 8% and y represent the amount she invests at 7%. Write an equation stating that Jane invests a total of \$14,000.

40. In exercise 39, Jane receives a total of \$1,000 from her investments. If income equals the amount invested times the rate of interest, write an equation relating Jane's total income to her two investments.

Review exercises

1. Given $2x + y = 3$, solve for x when $y = x + 2$. See sections 1–8 and 2–6.

Find the equation in standard form of each line having the given properties. See section 7–4.

2. Through $(-1,6)$, slope $\frac{2}{3}$

3. Through the points $(0,5)$ and $(-7,1)$

Factor the following expressions. See sections 4–3 and 4–4.

4. $16x^2 - 4y^2$

5. $9x^2 - 12x + 4$

6. $5y^2 - 6y - 8$

7. Find the solution set of the equation $4(3x - 2) + 2x = 6$. See section 2–6.

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8–3 ■ Solutions of systems of linear equations by substitution

A basic property of equality is that if two expressions are equal (represent the same quantity), one expression may replace the other expression in any equation to form an equivalent equation. We use this property to solve systems of equations by a third method, called the **substitution** method. This method is useful when

1. one of the equations is solved for one variable in terms of the other; for example, $y = 3x - 4$, or
2. one of the equations can *easily* be solved for one variable in terms of the other (usually when the coefficient of the variable is 1 or -1), for example, $x - 2y = 3$.

To illustrate, consider the system of linear equations

$$\begin{aligned} 3x + 2y &= 9 \\ y &= x + 2 \end{aligned}$$

Since the second equation is already solved for y in terms of x, we *substitute* $x + 2$ for y in the first equation.

$$\begin{aligned} 3x + 2y &= 9 \\ 3x + 2(x + 2) &= 9 && \text{Replace } y \text{ with } x + 2 \\ 3x + 2x + 4 &= 9 && \text{Solve for } x \\ 5x + 4 &= 9 \\ 5x &= 5 \\ x &= 1 \end{aligned}$$

Now substitute 1 for x in the second equation.

$$\begin{aligned} y &= x + 2 \\ y &= (1) + 2 && \text{Replace } x \text{ with } 1 \\ y &= 3 \end{aligned}$$

The solution is $(1,3)$. The solution set is $\{(1,3)\}$.

Note We can check our solution by substituting 1 for x and 3 for y in *both* original equations.

$$\begin{aligned} 3x + 2y &= 9 \\ 3(1) + 2(3) &= 9 \\ 3 + 6 &= 9 \\ 9 &= 9 \end{aligned} \qquad \begin{aligned} y &= x + 2 \\ (3) &= (1) + 2 \\ 3 &= 3 \end{aligned}$$

■ Example 8–3 A

Find the solution set of the following systems of linear equations by substitution.

1. $3x + 2y = 14$
$y = 2x - 7$

$$\begin{aligned} 3x + 2y &= 14 \\ 3x + 2(2x - 7) &= 14 && \text{Replace } y \text{ with } 2x - 7 \\ 3x + 4x - 14 &= 14 && \text{Multiply as indicated} \\ 7x - 14 &= 14 && \text{Combine like terms} \\ 7x &= 28 && \text{Add 14 to each member} \\ x &= 4 && \text{Divide each member by 7} \end{aligned}$$

Using the equation

$y = 2x - 7$
$y = 2(4) - 7$ Replace x with 4
$y = 8 - 7$
$y = 1$

The solution of the given system is (4,1). Check the solution. The solution set is $\{(4,1)\}$.

2. $2x - 4y = 7$
$x = 3y - 2$

$2x - 4y = 7$
$2(3y - 2) - 4y = 7$ Replace x with $3y - 2$
$6y - 4 - 4y = 7$ Multiply as indicated
$2y = 11$ Combine like terms; add 4 to each member
$y = \frac{11}{2}$ Divide each member by 2

Using the equation $x = 3y - 2$,

$x = 3\left(\frac{11}{2}\right) - 2$ Replace y with $\frac{11}{2}$
$x = \frac{33}{2} - 2$
$x = \frac{29}{2}$

The system has the solution $\left(\frac{29}{2},\frac{11}{2}\right)$. Check the solution. The solution set is $\left\{\left(\frac{29}{2},\frac{11}{2}\right)\right\}$.

3. $3x + y = 12$
$2x + 5y = 8$
To use substitution, we must first solve the first equation for y in terms of x.

$3x + y = 12$
$y = 12 - 3x$ Subtract $3x$ from each member

Using the equation $2x + 5y = 8$,

$2x + 5(12 - 3x) = 8$ Replace y with $12 - 3x$
$2x + 60 - 15x = 8$ Solve for x
$60 - 13x = 8$
$-13x = -52$
$x = 4$

Using the equation $y = 12 - 3x$,

$y = 12 - 3x$
$y = 12 - 3(4)$ Replace x with 4
$y = 12 - 12$ Multiply in right member
$y = 0$

The solution is the ordered pair (4,0). Check the solution. The solution set is $\{(4,0)\}$.

4. $8x + 2y = 6$
$y = -4x + 3$
Using the equation $8x + 2y = 6$,

$$8x + 2(-4x + 3) = 6 \quad \text{Replace } y \text{ with } -4x + 3$$
$$8x - 8x + 6 = 6$$
$$6 = 6 \quad \text{(True)}$$

Since $6 = 6$ is a true statement, the system is dependent and there are infinitely many solutions that satisfy the equations $8x + 2y = 6$ and $y = -4x + 3$.

5. $-4x + 2y = 7$
$y = 2x + 9$
Since we have $y = 2x + 9$, we substitute $2x + 9$ for y in the equation $-4x + 2y = 7$.

$$-4x + 2y = 7$$
$$-4x + 2(2x + 9) = 7 \quad \text{Replace } y \text{ with } 2x + 9$$
$$-4x + 4x + 18 = 7$$
$$18 = 7 \quad \text{(False)}$$

Since $18 = 7$ is a *false* statement, there are no solutions, and the system is inconsistent. The solution set is $\emptyset$ (the null set). Lines are parallel.

▶ ***Quick check*** Find the solution set of the system of linear equations using substitution. $3x - y = 7$
$4x - 5y = 2$

To solve a system of linear equations by substitution

1. Solve one of the equations for one of the variables in terms of the other (if this is not already done).
2. Substitute the expression obtained in step 1 into the other equation and solve.
3. Substitute the value obtained in step 2 into either equation and solve for the other variable.
4. The solution is the ordered pair obtained from steps 2 and 3.
5. If step 2 results in
   a. the variables being eliminated and a true statement is obtained, the system is dependent and there are infinitely many solutions (same line).
   b. the variables being eliminated and a false statement is obtained, the system is inconsistent and there are no simultaneous solutions (parallel lines).

As a final note, when the method of solution for a system of linear equations is not specified, the following are good criteria by which a method is selected.

1. The process of elimination is generally the easiest.
2. Substitution is most useful when the coefficient of one of the variables is 1 or -1 or an equation is solved for one variable in terms of the other variable.

3. Graphical solutions are approximations and can be used when exact answers are not necessary or to verify the results that we find algebraically.

Mastery points

Can you

- Solve systems of linear equations by substitution?
- Choose an appropriate method for solving a system of linear equations?

Exercise 8–3

Find the solution set of the following systems of linear equations by *substitution*. If a system is inconsistent or dependent, so state. See example 8–3 A.

Example $3x - y = 7$
$4x - 5y = 2$

Solution Solve $3x - y = 7$ for y. Then $y = 3x - 7$.

$$\begin{aligned} 4x - 5y &= 2 \\ 4x - 5(3x - 7) &= 2 \quad \text{Replace } y \text{ with } 3x - 7 \\ 4x - 15x + 35 &= 2 \quad \text{Solve for } x \\ -11x + 35 &= 2 \\ -11x &= -33 \\ x &= 3 \end{aligned}$$

Using the equation

$$\begin{aligned} 3x - y &= 7 \\ 3(3) - y &= 7 \quad \text{Replace } x \text{ with } 3 \\ 9 - y &= 7 \\ -y &= -2 \\ y &= 2 \end{aligned}$$

The solution is (3,2) and the solution set is $\{(3,2)\}$.

1. $3x - y = 10$; $y = -x + 2$
2. $2x + 3y = 9$; $x = y - 3$
3. $-2x + 5y = 17$; $y = 2x + 5$
4. $5x + y = 10$; $x = 2 - 3y$
5. $2y - x = 3$; $x = 4y - 1$
6. $3x + 4y = -1$; $y = 2 - 3x$
7. $4y - x = 3$; $x = 2y + 1$
8. $3y + 4x = -4$; $x = 4 - y$
9. $5x - 2y = 3$; $y = x - 4$
10. $3x + y = 10$; $2x + y = 5$
11. $2x + y = 13$; $3x + y = 17$
12. $3y + x = -10$; $3y - x = 4$
13. $2x - y = 2$; $6x - y = 22$
14. $x - y = 3$; $2x + 3y = 11$
15. $2x + y = 3$; $3x - y = 4$
16. $x + 5y = 7$; $2x + 3y = 5$
17. $5x - 3y = 4$; $x + 2y = -2$
18. $x + y = 4$; $3x - 5y = 7$
19. $x - y = 3$; $2x - 2y = 11$
20. $x + y = 5$; $3x + 3y = 3$
21. $3x - y = 7$; $6x - 2y = 14$
22. $-x + 3y = -3$; $9y - 3x = -9$
23. $3x + y = -3$; $x - 3y = -1$
24. $2x + y = 1$; $-3x + y = 1$

25. $2x + y = -3$
$5x - 4y = 1$

26. $5x + 2y = 11$
$7x - y = 4$

27. $2x + 3y = 2$
$6x - y = 5$

28. $x - 2y = 4$
$3x + 2y = 4$

29. $2x + y = 6$
$3x + 4y = 4$

30. $-x - 4y = 3$
$2x + 8y = -6$

31. $x + 4y = -3$
$-2x - 8y = 6$

32. $x - y = 7$
$3x + 3y = 4$

33. $5x - 3y = 11$
$x + y = 5$

34. $-x - 2y = 6$
$6x + 4y = 5$

35. $-3x - 2y = 6$
$6x + 4y = 5$

36. $4x + 3y = 1$
$y = -4$

37. $3x - 5y = -6$
$x = 5$

38. $-5x + 2y = 11$
$x = -3$

39. $3x - 7y = 14$
$y = 2$

40. $y = 2x + 1$
$y = -x + 3$

41. $y = 5x + 3$
$y = 2x - 4$

42. $y = 3x - 4$
$y = 3x + 5$

Find the solution set of each system of linear equations by either elimination or substitution. Try to choose the most suitable method.

43. $3x - 2y = -1$
$2x + 2y = 1$

44. $x - y = 2$
$-5x + 2y = -2$

45. $x - 3 = 0$
$3x + 2y = 1$

46. $y + 2x = 4$
$x = -6y + 1$

47. $\frac{1}{2}x - y = 3$
$2x + \frac{1}{3}y = 1$

48. $\frac{2x}{3} - \frac{y}{2} = 1$
$3x + \frac{y}{4} = 2$

49. $\frac{y}{3} - \frac{x}{4} = 3$
$\frac{x}{2} + \frac{y}{5} = -2$

50. $4x - 3y = -10$
$y + 2 = 0$

51. $4x - y = 8$
$5x + 3y = -4$

52. $7x - 2y = 3$
$4x + 3y = -2$

53. Let x represent one current in an electrical circuit and y represent a second current. If the first current has twice as many amperes as the second, write an equation stating this.

54. If the sum of the two currents in exercise 53 is 56 amperes, write an equation stating this.

55. A clothier has two kinds of suits. Let x represent the number of suits selling for one price and y represent the number of suits selling for another price. If he has a total of 80 suits, write an equation stating this.

56. In exercise 55, if the cost of the first kind of suit (x) is \$190 per suit and the cost per suit of the second kind (y) is \$250, write an equation stating that the total income from the suits was \$8,400.

Review exercises

1. A 42-foot piece of wood is cut into two pieces. If one piece is 6 feet less than twice the length of the other piece, how long are the pieces of wood? See section 2–8.

2. The length of a rectangle is 1 yard longer than twice its width. If the area of the rectangle is 36 square yards, find the length and width of the rectangle. See section 4–8. (*Hint:* Area = length × width.)

3. Solve the equation $A = \frac{1}{2}h(b + c)$ for b. See section 2–7.

4. Find the slope of the line through the points $(-4,4)$ and $(5,6)$. See section 7–3.

5. Solve the rational equation $\frac{4}{x} - \frac{3}{2x} = 4$. See section 6–5.

6. Solve the inequality $-4 \leq 1 - 5x < 6$. See section 2–6.

8–4 ■ Applications of systems of linear equations

In chapter 2, we learned how to take a verbal statement and translate it into an algebraic equation in one unknown. Many practical problems that can be solved using single equations in one unknown can more easily be solved by translating into *two* equations involving *two* unknowns.

While there is no standard procedure *for solving applied problems*, the following guidelines should be useful.

Solve a word problem using systems of linear equations

1. Read the problem carefully, noting the information given and what you are asked to find.
2. Determine any prior knowledge that may be useful in setting up the equations: formulas, distance-rate-time, interest, and so on. Draw a sketch, if appropriate.
3. Choose two variables to represent the unknown quantities.
4. Translate separate statements from the verbal statement of the problem into individual equations.
5. Solve the resulting system of linear equations by one of the methods we have studied.
6. Write the answer—usually a sentence containing the numbers from the simultaneous solution—using the correct units of measure. Check your results in the original statement of the problem.

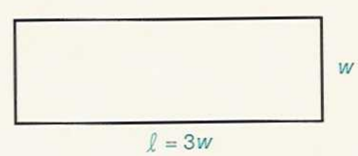
■ Example 8–4 A

1. The length of a rectangle is 3 times as long as its width. The perimeter of the rectangle is 40 inches. What are the dimensions of the rectangle?

Note We need the prior knowledge that the perimeter, P, of a rectangle is given by the formula $P = 2\ell + 2w$, where ℓ is the length and w is the width of the rectangle.

Let ℓ represent the length of the rectangle and w represent the width of the rectangle.

length	is	3 times the width
ℓ	$=$	$3w$



We use the formula for the perimeter, $P = 2\ell + 2w$, and since the perimeter is 40 inches, we have $40 = 2\ell + 2w$. Therefore, we have the system of linear equations

$$\begin{aligned} \ell &= 3w \\ 2\ell + 2w &= 40 \end{aligned}$$

Using substitution and the equation $2\ell + 2w = 40$,

$$\begin{aligned} 2(3w) + 2w &= 40 && \text{Replace } \ell \text{ with } 3w \\ 8w &= 40 && \text{Solve for } w \\ w &= 5 \end{aligned}$$

Substituting 5 for w in the equation $\ell = 3w$, we get

$\ell = 3(5) = 15$

The rectangle is 15 inches long and 5 inches wide.

Check:

a. The perimeter is 40 inches.

$$\begin{aligned} 2(15) + 2(5) &= 40 \\ 30 + 10 &= 40 \\ 40 &= 40 \quad \text{(True)} \end{aligned}$$

b. The length is 3 times the width.

$$\begin{aligned} 15 &= 3(5) \\ 15 &= 15 \quad \text{(True)} \end{aligned}$$

2. A woman has \$10,000, part of which she invests at 11% and the rest at 8% annual interest. If her total income for one year from the two investments is \$980, how much does she invest at each rate?

Note The prior knowlege needed for this problem is that Simple interest = Principal × Rate × Time. Time in this example is 1 year. When using this formula, time should be stated in years.

Let x = amount invested at 11% and y = amount invested at 8%.
Then, $(0.11)x$ = interest from the 11% investment for one year
$(0.08)y$ = interest from the 8% investment for one year

$$\underbrace{(0.11)x}_{\text{11\% investment}} \;\underbrace{+}_{\text{plus}}\; \underbrace{(0.08)y}_{\text{8\% investment}} \;\underbrace{=}_{\text{is}}\; \underbrace{980}_{\$980}$$

From "A woman has \$10,000 . . . ,"

$x + y = 10{,}000$

The system of linear equations is

$$\begin{aligned} x + y &= 10{,}000 \\ (0.11)x + (0.08)y &= 980 \end{aligned}$$

Multiply the second equation by 100 to clear the decimal fractions.

$$\begin{aligned} 100(0.11)x + 100(0.08)y &= 100(980) \\ 11x + 8y &= 98{,}000 \end{aligned}$$

We then have the system

$$\begin{array}{lcrcl} x + y = 10{,}000 & \text{Multiply each term by } (-8) \rightarrow & -8x - 8y &=& -80{,}000 \\ 11x + 8y = 98{,}000 & & 11x + 8y &=& 98{,}000 \\ \hline & & 3x &=& 18{,}000 \quad \text{Add} \\ & & x &=& 6{,}000 \quad \text{Divide by 3} \end{array}$$

Using $x + y = 10{,}000$,

$$\begin{aligned} (6{,}000) + y &= 10{,}000 \quad \text{Replace } x \text{ with 6,000} \\ y &= 4{,}000 \end{aligned}$$

The woman invested \$6,000 at 11% and \$4,000 at 8% interest.

Check:

a. The total yearly income is \$980.

$$\begin{aligned}(0.11)(6{,}000) + (0.08)(4{,}000) &= 980\\ 660 + 320 &= 980\\ 980 &= 980 \quad \text{(True)}\end{aligned}$$

b. The woman has \$10,000.

$$\begin{aligned}6{,}000 + 4{,}000 &= 10{,}000\\ 10{,}000 &= 10{,}000 \quad \text{(True)}\end{aligned}$$

3. The sum of two lengths of wire is 11 meters. If four times the first length is added to the second length, the result is 25 meters. Find the two lengths of wire.

Let x = the first length of wire and y = the second length of wire.

the sum of the two lengths	is	11 meters
$x + y$	$=$	11

four times the first length	added to	the second length	is	25 meters
$4x$	$+$	y	$=$	25

Therefore, we have the system of linear equations

$$\begin{aligned}x + y &= 11\\ 4x + y &= 25\end{aligned} \qquad \text{Multiply each term by } (-1) \qquad \begin{aligned}-x - y &= -11\\ 4x + y &= 25\\ \hline 3x &= 14 \quad \text{Add}\\ x &= \frac{14}{3} = 4\frac{2}{3}\end{aligned}$$

Using the equation $x + y = 11$, substitute $\frac{14}{3}$ for x and solve for y.

$$\begin{aligned}\left(\frac{14}{3}\right) + y &= 11 && \text{Replace } x \text{ with } \frac{14}{3}\\ y &= 11 - \frac{14}{3} && \text{Subtract } \frac{14}{3} \text{ from each member}\\ y &= \frac{33}{3} - \frac{14}{3} = \frac{19}{3} && \text{Change to common denominator 3 and subtract}\\ y &= 6\frac{1}{3}\end{aligned}$$

Thus the two wires have length $4\frac{2}{3}$ meters and $6\frac{1}{3}$ meters.

Check:

a. The sum of the two lengths is 11.

$$\begin{aligned}\frac{14}{3} + \frac{19}{3} &= 11\\ \frac{33}{3} &= 11\\ 11 &= 11 \quad \text{(True)}\end{aligned}$$

b. Four times the first length added to the second length is 25.

$$\begin{aligned} 4\left(\frac{14}{3}\right) + \frac{19}{3} &= 25 \\ \frac{56}{3} + \frac{19}{3} &= 25 \\ \frac{75}{3} &= 25 \\ 25 &= 25 \quad \text{(True)} \end{aligned}$$

4. What quantities of silver that are 65% pure and 45% pure must be mixed together to get 100 grams of silver that is 50% pure?

Note 65% pure silver means the metal is 65% silver and 35% of some other metal(s).

Let $x =$ the number of grams of 65% pure silver, and $y =$ the number of grams of 45% pure silver.

We can use the following table to summarize the given information.

Solution	**Amount of solution**	**Percent of silver**	**Amount of silver in solution**
First	x	65%	$0.65x$
Second	y	45%	$0.45y$
Final	100	50%	$0.50(100) = 50$

$x + y = 100$ $\qquad$ $0.65x + 0.45y = 50$

We must now solve the system of linear equations.

$$\begin{aligned} x + y &= 100 \\ 0.65x + 0.45y &= 50 \end{aligned}$$

Multiply the second equation by 100 to clear decimals.

$$\begin{aligned} -65x - 65y &= -6{,}500 \quad \text{Multiply } x + y = 100 \text{ by } -65 \\ 65x + 45y &= 5{,}000 \\ \hline -20y &= -1{,}500 \\ y &= 75 \end{aligned}$$

Using the equation,

$$\begin{aligned} x + y &= 100 \\ x + (75) &= 100 \quad \text{Replace } y \text{ with } 75 \\ x &= 25 \end{aligned}$$

Then 25 grams of 65% pure silver must be mixed with 75 grams of 45% pure silver to get 100 grams of 50% pure silver.

Check: 1. $25 + 75 = 100$ $\quad$ Total of 100 grams

2. $0.65(25) + 0.45(75) = 50$ $\quad$ Sum of mixes yields 50 grams

$$\begin{aligned} 16.25 + 33.75 &= 50 \\ 50 &= 50 \end{aligned}$$

5. Two automobiles start at town A and go in opposite directions. After traveling for three hours, they are 351 miles apart. If one automobile is averaging 13 miles per hour faster than the other, what is the average speed of each automobile in miles per hour?

Use the formula that relates distance, rate, and time, $d = rt$. Time $t = 3$ since each automobile travels 3 hours.

Let x = the average speed of the slower automobile and y = the average speed of the faster automobile.

We can use the following table to organize the information.

	r	**t**	**d**
Slower automobile	x	3	$3x$
Faster automobile	y	3	$3y$

$$\downarrow$$
$$3x + 3y = 351$$

From "If one automobile is averaging 13 miles per hour faster than the other," we get the equation

$$y = x + 13$$

Thus we determine the system to be

$$\begin{aligned} y &= x + 13 \\ 3x + 3y &= 351 \end{aligned}$$

Substitute $x + 13$ for y in the second equation.

$$\begin{aligned} 3x + 3(x + 13) &= 351 \qquad \text{Replace } y \text{ with } x + 13 \\ 3x + 3x + 39 &= 351 \\ 6x + 39 &= 351 \\ 6x &= 312 \\ x &= 52 \end{aligned}$$

Replace x by 52 in the equation.

$$\begin{aligned} y &= x + 13 \\ y &= (52) + 13 = 65 \qquad \text{Replace } x \text{ with } 52 \end{aligned}$$

The faster automobile is traveling at 65 mph and the slower automobile is traveling at 52 mph.

Check:

$$\begin{aligned} 3x + 3y &= 351 \\ 3(52) + 3(65) &= 351 \\ 156 + 195 &= 351 \\ 351 &= 351 \end{aligned} \qquad \begin{aligned} y &= x + 13 \\ (65) &= (52) + 13 \\ 65 &= 65 \end{aligned}$$

▶ ***Quick check*** Karen and her mother are in cities 400 miles apart. They meet at a location on a line between the two cities. If Karen drives 20 miles per hour faster than her mother, and they meet after 4 hours, how fast was each person driving? ■

Mastery points

Can you

- Set up and solve a system of linear equations in two variables given a word problem?

Exercise 8–4

Solve the following problems by setting up a system of two linear equations with two unknowns.

Example Karen and her mother are in cities 400 miles apart. They meet at a location on a line between the two cities. If Karen drives 20 miles per hour faster than her mother, and they meet after 4 hours, how fast was each person driving?

Solution Let x = the speed of Karen's automobile and y = the speed of her mother's automobile.

We use the formula distance (d) = rate (r) · time (t). Time t for each automobile is 4 since each drives 4 hours. The following chart compiles this information using $r \cdot t = d$.

	rate (r)	time (t)	distance (d)
Karen	x	4	$4x$
Mother	y	4	$4y$

Then, since they are 400 miles apart,

total distance = 400 miles

$$4x + 4y = 400$$
$$x + y = 100 \quad \text{Divide each term by 4}$$

Since Karen drives 20 miles per hour faster than her mother,

$$x = y + 20$$

We solve the system of linear equations.

$$x + y = 100$$
$$x = y + 20$$

Using substitution,

$$(y + 20) + y = 100 \quad \text{Replace } x \text{ with } y + 20 \text{ in } x + y = 100$$
$$2y + 20 = 100$$
$$2y = 80$$
$$y = 40$$

Since $x = y + 20$

$$= (40) + 20 \quad \text{Replace } y \text{ with } 40$$
$$= 60$$

Karen was driving 60 miles per hour and her mother was driving 40 miles per hour.

See example 8–4 A–1.

1. The perimeter of a rectangle is 36 meters and the length is 2 meters more than three times the width. Find the dimensions.

2. The perimeter of a room is 40 feet, and twice the length increased by three times the width is 48 feet. What are the dimensions?

3. The perimeter of a rectangle is 100 meters. The rectangle is 22 meters longer than it is wide. Find the dimensions.

4. A rectangular field has a length that is 21 yards more than twice the width. The perimeter is 620 yards. What are the dimensions of the field?

5. The length of a room is 2 feet less than twice its width. If its perimeter is 62 feet, what are the dimensions?

See example 8–4 A–2.

$\boxed{6.}$ Phil has \$20,000, part of which he invests at 8% interest and the rest at 6%. If his total income for one year from the two investments was \$1,460, how much did he invest at each rate?

7. Nancy has \$18,000. She invests part of her money at $7\frac{1}{2}$% interest and the rest at 9%. If her yearly income from the two investments was \$1,560, how much did she invest at each rate?

8. Rich has \$18,000, part of which he invests at 10% interest and the rest at 8%. If his yearly income from each investment was the same, what did he invest at each rate?

9. Dick has \$30,000, part of which he invests at 9% interest and the rest at 7%. If his yearly income from the 7% investment was \$820 more than that from the 9% investment, how much was invested at each rate?

10. Lynne made two investments totaling \$25,000. On one investment she made an 18% profit, but on the other investment she took an 11% loss. If her yearly net gain was \$2,180, how much was in each investment?

11. Anne made two investments totaling \$21,000. One investment made her a 13% profit, but on the other investment she took a 9% loss. If her net yearly loss was \$196, how much was in each investment?

See example 8–4 A–3.

$\boxed{12.}$ A piece of pipe is 19 feet long. The pipe must be cut so that one piece is 5 feet longer than the other piece. What are the lengths of the two pieces of pipe?

13. A 12-foot board is cut into two pieces so that one piece is 4 feet longer than the other. How long is each piece?

14. A 24-foot rope is cut into two pieces so that one piece is twice as long as the other. How long is each piece?

15. The sum of two currents is 80 amperes. If the greater current is 24 amperes more than the lesser current, find their values.

16. A 50-foot extension cord is cut into two pieces so that one piece is 12 feet longer than the other piece. How long is each piece?

17. The sum of the number of teeth on two gears is 64 and their difference is 12. How many teeth are on each gear?

18. Two gears have a total of 83 teeth. One gear has 15 less teeth than the other. How many teeth are on each gear?

19. Two electrical voltages have a total of 126 volts. If one voltage is 32 volts more than the other, find the voltages.

20. The sum of two voltages is 85 and their difference is 32. Find the voltages.

21. The sum of the resistances of two resistors is 24 ohms and their difference is 14 ohms. How many ohms are in each resistor?

22. The resistance of one resistor exceeds that of another resistor by 25 ohms and their sum is 67 ohms. How many ohms are in each resistor?

23. A clothing store sells suits at \$125 and \$185 each. The store owners observe that they sold 40 suits for a total of \$5,720. How many suits of each type did they sell?

24. Three times the number of bolts in a piece of machinery is 3 less than twice the number of spot welds, while seven times the number of bolts is 5 more than four times the number of spot welds. How many bolts and spot welds are there?

25. When two batteries are connected in series, we add the voltages together to get a total voltage of 27 V. When the batteries are connected in opposition, the resulting voltage, 5 V, is their difference. Find the voltages of the two batteries.

26. When two batteries are connected in series, the total voltage is 53 V. When the batteries are connected in opposition, the total voltage is 7 V. Find the voltage of the two batteries. (Refer to exercise 25.)

27. Three times the number of teeth on a first gear is 1 more than the number of teeth on a second gear. Also, five times the number of teeth on the first gear is 4 less than twice the number of teeth on the second gear. Find the number of teeth on each gear.

28. The specific gravity of an object is defined as its weight in air divided by the difference between its weight in air and its weight when submerged in water. If an object has a specific gravity of 3 and the sum of its weight in air and in water is 30 pounds, find its weight in air and its weight in water.

See example 8–4 A–4.

29. An auto mechanic has two bottles of battery acid. One contains a 10% solution and the other a 4% solution. How many cubic centimeters (cc) of each solution must be used to make 120 cubic centimeters of a solution that is 6% acid?

30. A metallurgist wishes to form 2,000 kilograms (kg) of an alloy that is 80% copper. This alloy is to be obtained by fusing some alloy that is 68% copper and some that is 83% copper. How many kilograms of each alloy must be used?

31. If a jeweler wishes to form 12 ounces of 75% pure gold from sources that are 60% and 80% pure gold, how much of each substance must be mixed together to produce this?

32. A chemist wishes to make 1,000 liters of a 3.5% acid solution by mixing a 2.5% solution with a 4% solution. How many liters of each solution are necessary?

33. A drum contains a mixture of antifreeze and water. If 6 liters of antifreeze are added, the mixture will be 90% antifreeze, but if 6 liters of water are added, the mixture will be 70% antifreeze. What is the percentage of antifreeze in the mixture presently in the drum?

34. A pharmacist is able to fill 200 3-grain and 2-grain capsules using 500 grains of a certain drug. How many capsules of each kind does he fill?

35. A solution that is 38% silver nitrate is to be mixed with a solution that is 3% silver nitrate to obtain 100 centiliters (cl) of solution that is 5% silver nitrate. How many centiliters of each solution should be used in the mixture?

36. A druggist has two solutions, one 60% hydrogen peroxide and the other 30% hydrogen peroxide. How many liters of each should be mixed to obtain 30 liters of a solution that is 40% hydrogen peroxide?

See example 8–4 A–5.

37. Two cars are 100 miles apart. If they drive toward each other they will meet in 1 hour. If they drive in the same direction they will meet in 2 hours. Find their speeds.

38. A cyclist and a pedestrian are 20 miles apart. If they travel toward each other, they will meet in 75 minutes, but if they travel in the same direction, the cyclist will overtake the pedestrian in 150 minutes. What are their speeds?

39. Jane and Jim leave from a drugstore at the same time, walking in opposite directions. After 1 hour they are 9,680 yards apart. If Jim walked at a rate of 0.5 mph faster than Jane, how fast was each walking in miles per hour?
(*Hint:* 1 mile = 1,760 yards.)

40. A boat can travel 24 miles downstream in 2 hours and 16 miles upstream in the same amount of time. What is the speed of the boat in still water and what is the speed of the current? (*Hint:* Add the speed of the current when going downstream and subtract the speed of the current when going upstream.)

41. An airplane can fly at 460 miles per hour with the wind and 322 miles per hour against the wind. What is the speed of the wind and what is the speed of the airplane in still air?

42. If a boat takes $1\frac{1}{2}$ hours to go 12 miles downstream and 6 hours to return, what is the speed of the current and of the boat in still water?

Example A line whose equation is of the form $y = mx + b$ passes through a point (x,y) if and only if the coordinates of the point satisfy the equation. Find the values of m and b so that the line contains the points $(-1,-7)$ and $(2,5)$. Write the equation of the line.

Solution The ordered pairs $(-1,-7)$ and $(2,5)$ are solutions of the equation. We then substitute these values for x and y into $y = mx + b$ to obtain two equations in the variables m and b.

1. When $(x,y) = (-1,-7)$

$$y = mx + b$$
$$(-7) = m(-1) + b \quad \text{Replace } y \text{ with } -7 \text{ and } x \text{ with } -1$$
$$-7 = -m + b$$
$$-m + b = -7$$
$$m - b = 7 \quad \text{Write in standard form}$$

2. When $(x,y) = (2,5)$

$$y = mx + b$$
$$(5) = m(2) + b \quad \text{Replace } y \text{ with } 5 \text{ and } x \text{ with } 2$$
$$5 = 2m + b$$
$$2m + b = 5 \quad \text{Write in standard form}$$

Then we have the system of linear equations.

$$\begin{aligned} m - b &= 7 \\ 2m + b &= 5 \\ \hline 3m \quad &= 12 \quad \text{Add the equations} \\ m &= 4 \end{aligned}$$

Substitute 4 for m in $2m + b = 5$.

$$\begin{aligned} 2(4) + b &= 5 \quad \text{Replace } m \text{ with } 4 \\ 8 + b &= 5 \\ b &= -3 \end{aligned}$$

Replace b with -3 and m with 4 in $y = mx + b$ and the equation of the line containing the two points is $y = 4x - 3$.

$\boxed{43.}$ The line with equation $y = mx + b$ passes through a point (x,y) if and only if the coordinates of the point satisfy the equation. Find the values of m and b so that the line will contain the points $(1,-1)$ and $(2,2)$. Write the equation of the line.

44. Find the values of m and b for the line that passes through the points $(2,1)$ and $(-1,7)$. Write the equation of the line. (Refer to exercise 43.)

45. Find the values of m and b for the line that passes through the points $(-2,3)$ and $(3,-7)$. Write the equation of the line. (Refer to exercise 43.)

46. Find the values of m and b for the line that passes through the points $(-1,7)$ and $(2,-2)$. Write the equation of the line. (Refer to exercise 43.)

47. Find the values of m and b and write the equation of the line that passes through $(0,-4)$ and $(5,0)$. (Refer to exercise 43.)

48. Find the values of m and b and write the equation of the line that passes through $(0,0)$ and $(-4,5)$. (Refer to exercise 43.)

Review exercises

Perform the indicated operations. Express your answer with positive exponents. Assume all variables are nonzero. See sections 3–3 and 3–4.

1. $(-4x^2y^3)$

2. $\dfrac{x^{-2}y^3}{xy^2}$

3. $4^{-1} + 3^{-1}$

See sections 2–3 and 3–2.

4. $(2x^2 - x + 3) - (x^2 + 4x - 1)$

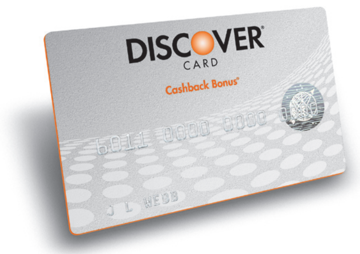
5. $4x^2(3x^2 + 2x - 3)$

6. $(3y + 2)(3y - 2)$

7. $(4x - 5y)^2$

8. Find the solution set of the equation $5(x - 2) = -3(2 - 3x)$. See section 2–6.

9. Find the solution set of the quadratic equation $x^2 - 3x - 10 = 0$. See section 4–7.



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8–5 ■ Solving systems of linear inequalities by graphing

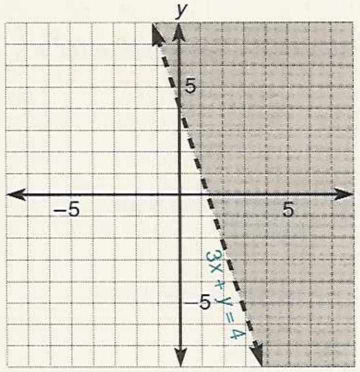


Figure 8–1

In section 7–5, we learned how to graph a linear inequality in two variables. To review this method, let us consider the inequality $3x + y > 4$. We first replace the inequality symbol, $>$, with the equal symbol, $=$, and graph the resulting equation $3x + y = 4$. Since we have the symbol $>$, the coordinates of the points on the line do not satisfy the inequality and the line should be dashed. We then choose a point on either side of the line, usually (0,0), and substitute its coordinates into the inequality.

$$3x + y > 4$$
$$3(0) + (0) > 4$$
$$0 > 4 \quad \text{(False)}$$

We then shade the half-plane that does *not* contain (0,0) to represent the graph of the inequality. (See figure 8–1.)

In this section, we will consider the graphical method of obtaining the solution of systems of linear inequalities such as

$$2x + y > 3$$
$$x - y > 1$$

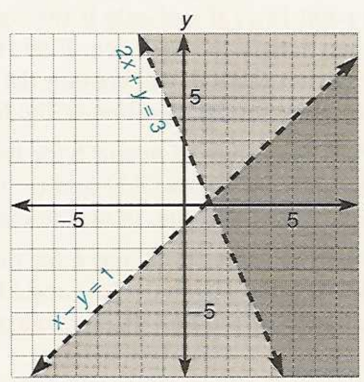
A system of linear inequalities, like a system of linear equations, has as its solution set the set of all ordered pairs of real numbers that are solutions of the inequalities of the system. The graph of a system of linear inequalities is the *intersection* of the graphs of the individual inequalities that make up the system.

■ Example 8–5 A

Graph the solution of the following systems of linear inequalities.

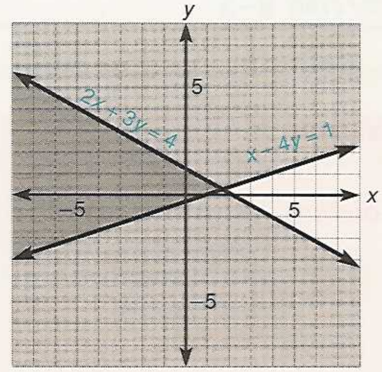
1. $2x + y > 3$
$x - y > 1$

First, graph each inequality on the same axes, shading each graph. (See the diagram.) The solution of the system is the overlap of the regions of the two graphs—the double-shaded region.



Note The dashed lines that show the ordered pairs of numbers for the points on the lines *are not solutions.*

2. $2x + 3y \leq 4$
$x - 4y \leq 1$
Graph $2x + 3y \leq 4$ and $x - 4y \leq 1$ on the same axes. The graph of the solution of the system is the overlapping (double-shaded) region of the diagram.



Note The symbol, $\leq$, involved in each inequality indicates that the points of that portion of the line bordering the double-shaded region are included as solutions. Therefore when graphing the equations, make the lines *solid*.

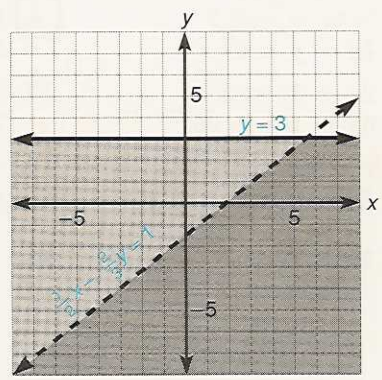
The solutions are in the shaded region and *include both* half-lines that form the boundary of this region.

3. $\frac{1}{2}x - \frac{2}{3}y > 1$
$y \leq 3$

Graph the inequalities $\frac{1}{2}x - \frac{2}{3}y > 1$ and $y \leq 3$ on the same axes. First, clear the denominators in the first inequality by multiplying by the LCD, 6.

$$6 \cdot \frac{1}{2}x - 6 \cdot \frac{2}{3}y > 6 \cdot 1$$
$$3x - 4y > 6$$

Recall, the graph of $y = 3$ is a horizontal line passing through the point (0,3). The graph of the solutions is the double shading in the diagram, together with that portion of the line $y = 3$ that borders the double-shaded region.



Note The point of intersection (6,3), of the dashed line and the solid line is *not* in the solution.

▶ ***Quick check*** Graph the solution of the system of linear inequalities.
$2x + y \geq 1$
$2y < x + 2$

Mastery points

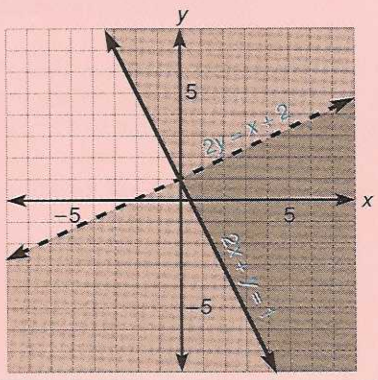
Can you
- Solve systems of linear inequalities by graphing?

Exercise 8–5

Graph the solution set of the following systems of linear inequalities. See example 8–5 A.

Example $2x + y \geq 1$
$2y < x + 2$

Solution Graph $2x + y \geq 1$ and $2y < x + 2$ on the same axes. A solid line is used in the graph of the first inequality and a dashed line is used in the graph of the second inequality. The graph is the double-shaded portion of the diagram, together with that portion of the line $2x + y = 1$ bordering the double-shaded region.



1. $x + y < 2$
   $x - y < 4$
2. $x + y < 3$
   $x - y > 1$
3. $2x + y > 1$
   $x - 2y > 2$
4. $x + 3y > 4$
   $2x - y < 0$
5. $-4x + y < 5$
   $2x - y < 0$
6. $5x - y < 5$
   $2x + y \geq 1$
7. $2x + 3y > 5$
   $3x - 2y < 4$
8. $4x + 3y > 12$
   $5x - 3y > 15$
9. $2x + 3y < 5$
   $8x + 10y > 26$
10. $5x - 2y > 15$
    $2x - 3y < 6$
11. $3x - 2y \geq 1$
    $2x \geq 5y$
12. $x - 4y \geq 3$
    $3y \leq 2x$
13. $y \leq 2x - 7$
    $2y + x \geq 5$
14. $4x + 5y \geq 20$
    $y \geq x + 1$
15. $5y + 2x \geq 3$
    $x \leq 2y - 1$
16. $x + y \geq 5$
    $x \geq 3y + 2$
17. $2x - 3y \geq 1$
    $-4x + 6y > -3$
18. $-4x + 10y \leq 5$
    $-2x + 5y < -1$
19. $\frac{1}{3}x - y \geq 2$
    $\frac{1}{3}x + 2y \leq 4$
20. $\frac{3}{4}x - \frac{2}{3}y \leq 1$
    $2x + \frac{1}{2}y \geq 3$
21. $\frac{1}{5}x \geq 3y + 1$
    $x - \frac{2}{3}y \leq 2$
22. $\frac{3}{2}y - 4x \geq 2$
    $\frac{1}{3}y > 1 - x$
23. $3x - 5y > 2$
    $x \leq 3$
24. $x - 4y \geq 6$
    $x > -2$
25. $5x - 2y \leq 0$
    $y > -1$
26. $2y - x > 1$
    $y \leq 3$
27. $x \geq -2$
    $y < 4$
28. $x \leq 5$
    $y > -3$
29. $x \geq 0$
    $y \geq 0$
30. $x \leq 0$
    $y \geq 0$

Note Exercises 29 and 30 define quadrants I and II, respectively, together with a portion of each axis.

31. State systems of linear inequalities that define (a) quadrant III, (b) quadrant IV.

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Review exercises

Perform the indicated operations. See sections 2–3, 3–2, and 3–4.

1. $(3x^2 - 2x + 1) - (x^2 - 5x - 6)$ **2.** $(4x + 1)(x - 3)$

3. $(5y - 2)^2$ **4.** $(7z + 1)(7z - 1)$

5. $a^3a^2a^{-1}$ **6.** $\dfrac{x^2y^3}{xy}$ **7.** $(-y^3)^3$

Factor the following expressions. See sections 4–1 and 4–3.

8. $5x^2 - 3x$ **9.** $4x^2 + 8x - 5$ **10.** $9y^2 - 30y + 25$

Chapter 8 lead-in problem

A keypunch operator at a local firm works for \$9 per hour while an entry-level typist works for \$6.50 per hour. The total pay for an 8-hour day is \$476. If there are two more typists than keypunch operators, how many keypunch operators does the firm employ?

Solution

Let x = the number of keypunch operators and y = the number of entry-level typists.
From "there are two more typists than keypunch operators,"

$$y = x + 2$$

In an 8-hour day, the pay for each

$$\begin{aligned} \text{keypunch operator is } 8 \cdot 9 &= \$72 \\ \text{typist is } 8 \cdot \$6.50 &= \$52 \end{aligned}$$

The table shows the pay for all the workers.

	Number	Pay per worker	Total pay
Keypunch operators	x	72	$72x$
Typists	y	52	$52y$

From "The total pay for an 8-hour day is \$476,"

$$72x + 52y = 476$$

We must solve the system of linear equations.

$$\begin{aligned} y &= x + 2 \\ 72x + 52y &= 476 \end{aligned}$$

Using substitution, substitute $x + 2$ for y in the second equation.

$$\begin{aligned} 72x + 52y &= 476 \\ 72x + 52(x + 2) &= 476 && \text{Replace } y \text{ with } x + 2 \\ 72x + 52x + 104 &= 476 && \text{Distribute in left member} \\ 124x + 104 &= 476 && \text{Combine in left member} \\ 124x &= 372 && \text{Subtract 104 from each member} \\ x &= 3 && \text{Divide each member by 124} \end{aligned}$$

There are 3 keypunch operators working for the firm.

Chapter 8 summary

1. A **system of linear equations** consists of two or more linear equations in the same variables. We have limited the number of variables to two.
2. A **simultaneous solution** of a system of two linear equations is the ordered pair (x,y) that is a solution of both of the equations.
3. A **graphical solution** to a system of two linear equations is found by estimating the coordinates of the point of intersection.
4. An **inconsistent** system of equations has no solution.
5. A **dependent** system of equations has an unlimited number of solutions.
6. A system of linear equations can be solved by **elimination** or **substitution** when an exact answer is required.
7. **Systems of linear inequalities** can be solved by graphing. The graph will consist of all points in a region of the plane bounded by the two lines.

Chapter 8 error analysis

1. Solving systems of linear equations
   Example: Solve the system.
   $$\begin{array}{lrl} 2x - 3y = -1 & & 2x - 3y = -1 \\ x + 4y = -2 & \text{Multiply by 2} \longrightarrow & \underline{2x + 8y = -4} \\ & \text{Add} & 4x + 5y = -5 \end{array}$$
   Correct answer:
   $$\begin{array}{lrl} 2x - 3y = -1 & & 2x - 3y = -1 \\ x + 4y = -2 & \text{Multiply by } -2 \longrightarrow & \underline{-2x - 8y = 4} \\ & \text{Add} & -11y = 3 \\ & & y = -\frac{3}{11} \end{array}$$
   What error was made? (*see page 342*)
2. Solutions of systems of equations
   Example: (2,3) is the solution of the system
   $4x - 3y = -1$
   $2x + y = 6$
   Check: $4(2) - 3(3) = -1$
   $8 - 9 = -1$
   $-1 = -1$
   (2,3) is the solution.
   Correct answer: (2,3) is not a solution of $2x + y = 6$ so it is not a solution of the system.
   What error was made? (*see page 336*)
3. Subtracting real numbers
   Example: $(-10) - (-5) = -15$
   Correct answer: -5
   What error was made? (*see page 41*)
4. Division of real numbers
   Example: $\frac{-16}{-4} = -4$
   Correct answer: $\frac{-16}{-4} = 4$
   What error was made? (*see page 52*)
5. Product of a monomial and a multinomial
   Example: $-4y(y^2 + 3y - 4) = 4y^3 + 12y^2 - 16y$
   Correct answer: $-4y^3 - 12y^2 + 16y$
   What error was made? (*see page 134*)
6. Dividing unlike bases
   Example: $\frac{x^3}{y} = x^2$
   Correct answer: $\frac{x^3}{y} = \frac{x^3}{y}$
   What error was made? (*see page 140*)
7. Zero exponent
   Example: $4y^0 = 1$
   Correct answer: 4
   What error was made? (*see page 142*)
8. Scientific notation
   Example: $-4.21 \times 10^2 = 421$
   Correct answer: -421
   What error was made? (*see page 149*)
9. Factoring the sum of two squares
   Example: $25a^2 + 36b^2 = (5a + 6b)(5a - 6b)$
   Correct answer: $25a^2 + 36b^2$ cannot be factored.
   What error was made? (*see page 176*)
10. Completely factoring the difference of two squares
    Example: $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$
    Correct answer: $(x^2 + y^2)(x + y)(x - y)$
    What error was made? (*see page 177*)

Chapter 8 critical thinking

Pick any integer from 20 to 29 and subtract the sum of its digits from it. The answer is always 18. Why is this true?

Chapter 8 review

[8–1]

Find the solution set of the following systems of linear equations by graphing. All answers in the back of the book will be given exactly. If the system is inconsistent or dependent, so state.

1. $x + y = 4$
$x - y = 2$

2. $3x - y = 2$
$x + y = 6$

3. $x - 2y = 0$
$2x - y = 6$

4. $x + y = 4$
$-x + y = 8$

5. $x - \frac{1}{3}y = 1$
$2x - \frac{2}{3}y = -2$

6. $x - y = 4$
$-3x + 3y = -12$

[8–2]

Find the solution set of the following systems of linear equations by elimination. If the system is inconsistent or dependent, so state.

7. $x - 2y = 3$
$x + y = 3$

8. $x - y = 5$
$3x + 2y = 25$

9. $2x + 3y = 1$
$-x + y = 2$

10. $5x + y = 4$
$5x - 3y = 8$

11. $x + 3y = 6$
$2x - 3y = -6$

12. $\frac{1}{2}x + \frac{1}{4}y = \frac{7}{4}$
$\frac{2}{3}x - \frac{1}{3}y = \frac{13}{3}$

13. $(0.3)x - (0.2)y = 1.1$
$(0.3)x + (0.2)y = 1.9$

14. $x - 2y = 5$
$-3x + 6y = 4$

15. $2x - y = 3$
$3x + y = 7$

16. $3x - y = 2$
$9x - 3y = 6$

[8–3]

Find the solution set of the following systems of linear equations by substitution. If the system is inconsistent or dependent, so state.

17. $x - y = 2$
$2x - 3y = 1$

18. $2x + y = 7$
$2x - y = 13$

19. $3x - 2y = 4$
$6x - 4y = 5$

20. $2x - 3y = 3$
$3x + y = 7$

21. $x + y = 4$
$x - y = 2$

22. $x - y = 5$
$\frac{1}{2}x + \frac{1}{3}y = \frac{25}{6}$

23. $12x - 9y = 15$
$4x - 3y = 5$

24. $5x + 3y = 4$
$2x - y = 5$

[8–4]

Solve the following problems by setting up a system of two linear equations with two unknowns.

25. The perimeter of a rectangle is 64 feet. Find the dimensions if three times the width is 4 less than the length.

26. Bruce made two investments totaling \$18,000. On one investment he made a 14% profit, but on the other investment he took a 23% loss. If his net yearly loss was \$70, how much was in each investment?

27. Four times the number of teeth on a first gear is 6 less than the number of teeth on a second gear. If the total number of teeth on both gears is 66, how many teeth are on each gear?

28. A chemist wishes to make 120 milliliters (ml) of a 75% acid solution by mixing a 60% solution with an 80% solution. How many milliliters of each solution are necessary?

[8–5]

Solve the following systems of linear inequalities by graphing.

29. $x + y \geq 5$
$2x - y < 3$

30. $4x - 3y < 1$
$2x + 5y \leq 2$

31. $4y - 3x \geq 2$
$y \leq 1 - 2x$

32. $2y \leq 4 - x$
$5y - x > 2$

33. $x > 4y + 3$
$y \leq 4$

34. $x > 1$
$5y - x \leq 0$

Chapter 8 cumulative test

Evaluate each of the following.

[3–3] 1. -4^{-3}

[3–3] 2. $\left(\frac{3}{4}\right)^{-1}$

[3–3] 3. $\left(\frac{3}{5} + \frac{1}{8}\right)^{0}$

[3–3] 4. $(-5)^{-2}$

[3–3] 5. $\frac{1}{2^{-4}}$

[2–2] 6. Evaluate the expression $\frac{ab - bc}{a}$, when $a = \frac{1}{2}$, $b = \frac{2}{3}$, and $c = \frac{3}{4}$.

Completely factor the following expressions.

[4–1] 7. $3a^3 + 15a^2 + 27a$

[4–4] 8. $x^2 - 121$

[4–1] 9. $ax + ay - 5x - 5y$

[4–3] 10. $3a^2 - 5ab - 2b^2$

[4–4] 11. $7x^3 - 7x$

[4–3] 12. $5b^2 - 22b + 8$

Perform the indicated operations and simplify the results.

[6–1] 13. $\frac{4x}{5y} \div \frac{24x^2}{15y^3}$

[6–3] 14. $\frac{5}{x^2 - 9x} - \frac{3}{x}$

[3–4] 15. $(-9x^2y^3z^3)^2$

[5–3] 16. $\frac{36x^3y^2 - 56xy^3}{4xy}$

[6–1] 17. $\frac{a^2 - 8a}{a^2 - a - 56} \cdot \frac{a^2 - 49}{7a}$

[3–2] 18. $(5x - 3y)^2$

[5–3] 19. $(8y^2 + 10y - 42) \div (4y - 7)$

Find the solution set of the following equations.

[6–5] 20. $\frac{4a - 1}{4} = \frac{5a + 2}{7}$

[4–7] 21. $x^2 = -9x$

[6–5] 22. $\frac{3}{4x} - 2 = \frac{5}{3x} + 1$

[4–7] 23. $7x^2 + 4x = 3$

Evaluate the following.

[1–3] 24. $|-6|$

[1–3] 25. $\left|\frac{4}{5}\right|$

[1–3] 26. $-\left|-\frac{1}{3}\right|$

Solve the following systems of linear equations by the appropriate method. If the system is inconsistent or dependent, so state.

[8–3] 27. $3y - x = 2$
$8y + x = 20$

[8–3] 28. $4x - y = 6$
$3x + 2y = -1$

[8–3] 29. $5y - 2z = 4$
$y = -3z + 1$

[8–3] 30. $x - 2y = -6$
$3x - 6y = 1$

[8–3] 31. $8x - 3y = 2$
$y - 1 = 0$

[8–3] 32. $y + 3x = 8$
$6x + 3y = 24$

[8–5] 33. Solve the system of linear inequalities by graphing.
$3x + 2y \leq 6$
$x - 5y > 5$

[8–4] 34. The sum of two numbers is 25. If one number is 1 more than three times the other, what are the numbers? (Solve by system of linear equations.)

[8–4] 35. The sum of two numbers is 79. The difference between the two numbers is 5. Find the numbers. (Solve by system of linear equations.)

[4–8] 36. The sum of the squares of two consecutive integers is 61. What are the numbers?

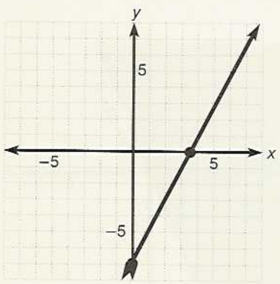
[4–8] 37. The square of a number is equal to five times that number. Find the number.

[6–6] 38. The sum of a number and twice its reciprocal is 3. Find the number.

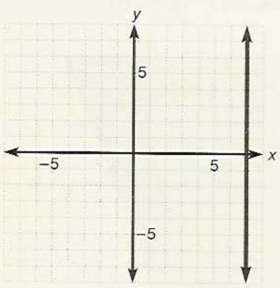
[6–6] 39. A tank can be filled in four hours by pipe A and in five hours by pipe B. How long will it take to fill the tank if both pipes are open?

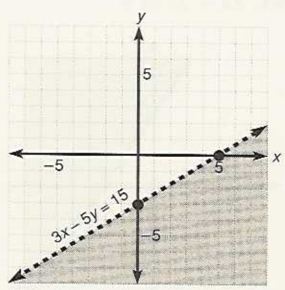
[2–8] 40. The length of a rectangle is 1 inch less than twice its width. If the perimeter is 40 inches, what are the dimensions of the rectangle?

9. $y^6(y^2 + y - 1)$ **10.** $p(p^3 - 4)(p + 1)$
11. $16(a + b)(a - b)$ **12.** $t(t - 7)(t - 1)$ **13.** $-7(5y + 4z)$
14. $16y^2 - 24xy + 9x^2$ **15.** $20x^2 - 3x - 56$ **16.** $25 - \frac{9}{16}y^2$
17. $x^3 - 3x^2y + 3xy^2 - y^3$ **18.** $15y^5 + 18y^4 + 32y^3 + 66y^2 - 7y + 56$ **19.** $\frac{5x^2 + 32x}{(x - 7)(x + 7)(x + 6)}$ **20.** $\frac{a^2 - 2a + 1}{(a + 5)(a - 2)}$
21. $\frac{6}{y + 3}$ **22.** $\frac{1}{x^9}$ **23.** $\frac{1}{8^8}$ **24.** $-\frac{125}{a^9}$ **25.** 7.76×10^{-6}
26. $3x^2 - 8x + 20 + \frac{-45}{x + 2}$ **27.** $\frac{2x^2 - x + 9}{3x + 4}$
28. x-intercept, $\frac{7}{2}$; y-intercept, -7



29. x-intercept, 7; y-intercept, none **30.**





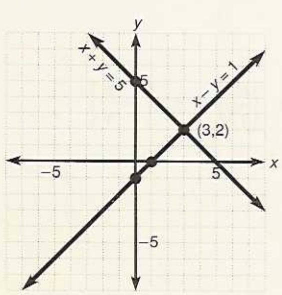
31. $m = \frac{7}{3}$ **32.** $3x - 4y = -13$ **33.** $3x + 5y = 27$
34. parallel **35.** neither **36.** $f(0) = -2, f(-3) = -11, f(4) = 10$ **37.** 136 **38.** 12 **39.** legs are 5 inches and 12 inches

Chapter 8

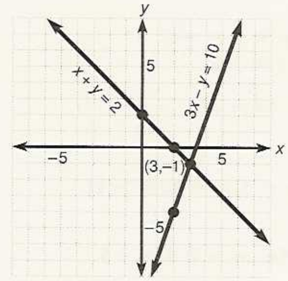
Exercise 8–1

Answers to odd-numbered problems

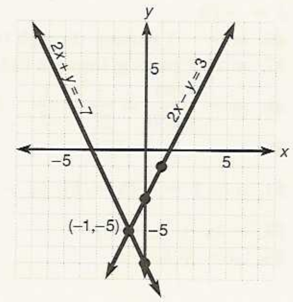
1. yes **3.** yes **5.** yes **7.** no **9.** yes **11.** no
13. solution: (3,2)



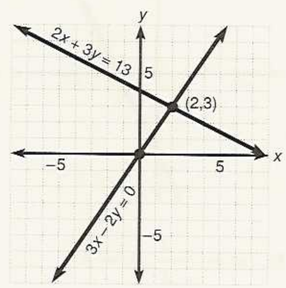
15. solution: (3,−1)



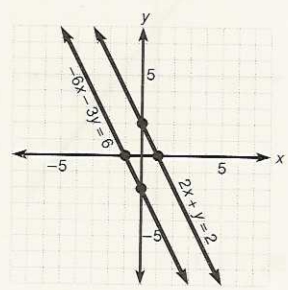
17. solution: (−1,−5)



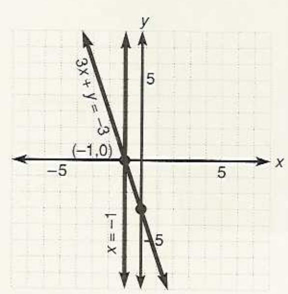
19. solution: (2,3)



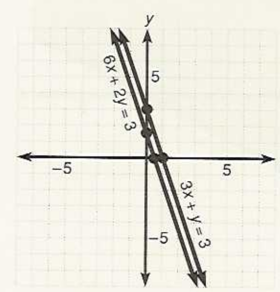
21. parallel lines (the slopes of both lines are the same) ∴ *inconsistent* with no solution



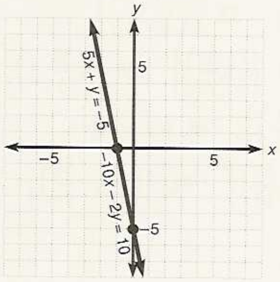
23. solution: (−1,0)



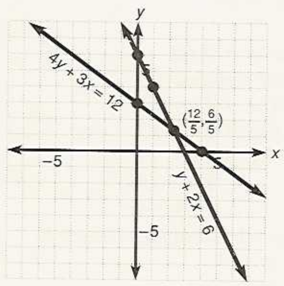
25. inconsistent; no solution



27. Both equations define the same line ∴ *dependent* with an unlimited number of solutions



29. solution: $\left(\frac{12}{5},\frac{6}{5}\right)$

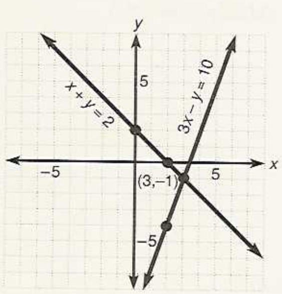


Solutions to trial exercise problems

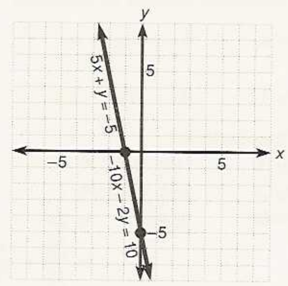
5. $2x + y = 2$
$6x - y = 22$ $\quad(3,-4)$
Let $x = 3$ and $y = -4$.
$2(3) + (-4) = 2$ $\quad$ $6(3) - (-4) = 22$
$6 - 4 = 2$ $\quad$ $18 + 4 = 22$
$2 = 2$ (true) $\quad$ $22 = 22$ (true)
Therefore $(3,-4)$ is a simultaneous solution.

10. $y = 4x - 3$
$y = -1$ $\quad(-1,-1)$
Let $x = -1$ and $y = -1$.
$-1 = 4(-1) - 3$ $\quad$ $y = -1$
$-1 = -4 - 3$ $\quad$ $-1 = -1$ (true)
$-1 = -7$ (false)
Therefore $(-1,-1)$ is *not* a simultaneous solution.

15. $3x - y = 10$
$x + y = 2$
$a_1 = 3, b_1 = -1, c_1 = 10$
$a_2 = 1, b_2 = 1, \quad c_2 = 2$
Since $\frac{3}{1} \neq \frac{-1}{1}$ the system is independent and consistent with simultaneous solution $(3,-1)$.



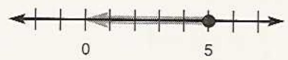
27. The system is *dependent* so all points in the line represent simultaneous solutions. (*Note:* If we divide the members of $-10x - 2y = 10$ by -2, we get equation $5x + y = -5$.)



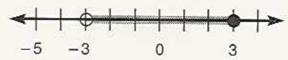
Review exercises

1. $\left\{\frac{4}{5}\right\}$ **2.** $3x^2 - 10x + 30 + \frac{-89}{x+3}$

3. $y \leq 5$



4. $-3 < x \leq 3$



5. x^4 **6.** $x^5 - 2x^4 + x^3$ **7.** $4x^2 - 4xy + y^2$

Exercise 8–2

Answers to odd-numbered problems

1. $\{(3,2)\}$ **3.** $\left\{\left(\frac{5}{3},-5\right)\right\}$ **5.** inconsistent (no solution) **7.** $\left\{\left(\frac{5}{3},2\right)\right\}$ **9.** $\{(-2,3)\}$ **11.** $\{(-1,3)\}$ **13.** $\left\{\left(\frac{29}{10},-\frac{3}{5}\right)\right\}$ **15.** dependent (unlimited number of solutions) **17.** $\{(1,-5)\}$ **19.** dependent (unlimited number of solutions) **21.** $\left\{\left(\frac{17}{21},-\frac{11}{21}\right)\right\}$ **23.** $\left\{\left(\frac{22}{21},\frac{13}{21}\right)\right\}$ **25.** $\{(5,-2)\}$ **27.** $\left\{\left(\frac{4}{5},\frac{9}{5}\right)\right\}$ **29.** inconsistent (no solution) **31.** $\{(-3,7)\}$ **33.** $(7,3)$ **35.** $y = 5x$ **37.** $y = x + 20$ **39.** $x + y = 14{,}000$

Solutions to trial exercise problems

6. $-x + y \ = 6$
$x + 4y = 4$

$$\begin{array}{r} -x + y = 6 \\ \underline{x + 4y = 4} \\ 5y = 10 \\ y = 2 \end{array}$$
Add the equations.

Substitute 2 for y in $-x + y = 6$. $-x + (2) = 6$
$-x = 4$ so $x = -4$
The solution is $(-4,2)$.
The solution set is $\{(-4,2)\}$.

10. $x - y = 3$ $\quad$ Multiply equation one by 3.
$\underline{2x + 3y = 11}$
$3x - 3y = 9$
$\underline{2x + 3y = 11}$
$5x \quad = 20$ $\quad$ Add the equations.
$x = 4$
Substitute 4 for x in $2x + 3y = 11$.
$2(4) + 3y = 11$
$8 + 3y = 11$
$3y = 3$
$y = 1$
The solution is $(4,1)$.
The solution set is $\{(4,1)\}$.

19. $-6x + 3y = 9$ $\qquad$ $-6x + 3y = 9$
$2x - y = -3$ Multiply by 3. $\quad$ $\underline{6x - 3y = -9}$
$0 = 0$

The system is dependent so an unlimited number of solutions.

27. $\frac{1}{2}x + \frac{1}{3}y = 1$ Multiply by 6. $\quad 3x + 2y = 6$

$\frac{2}{3}x - \frac{1}{4}y = \frac{1}{12}$ Multiply by 12. $\quad 8x - 3y = 1$

(*Note:* We have just cleared denominators.)

$3x + 2y = 6$ Multiply by 3. $\quad 9x + 6y = 18$
$8x - 3y = 1$ Multiply by 2. $\quad \underline{16x - 6y = 2}$
Add: $\quad 25x \quad = 20$

$$x = \frac{20}{25} = \frac{4}{5}$$

Substitute $\frac{4}{5}$ for x in $\frac{1}{2}x + \frac{1}{3}y = 1$.

$$\frac{1}{2}\left(\frac{4}{5}\right) + \frac{1}{3}y = 1$$
$$\frac{2}{5} + \frac{1}{3}y = 1$$
$$\frac{1}{3}y = \frac{3}{5}$$
$$y = \frac{9}{5}$$

The solution is $\left(\frac{4}{5},\frac{9}{5}\right)$.

The solution set is $\left\{\left(\frac{4}{5},\frac{9}{5}\right)\right\}$.

34. $x + (0.4)y = 3.4$ Multiply by 10. $\quad 10x + 4y = 34$
$(0.6)x - (1.4)y = 0.4$ Multiply by 10. $\quad 6x - 14y = 4$
(*Note:* The coefficients are now integers.)
$10x + 4y = 34$ Multiply by 3. $\rightarrow$ $\quad 30x + 12y = 102$
$6x - 14y = 4$ Multiply by -5. $\rightarrow$ $\quad \underline{-30x + 70y = -20}$
$82y = 82$
$y = 1$

Substitute 1 for y in $x + 0.4y = 3.4$.
$x + (0.4)(1) = 3.4$
$x + 0.4 = 3.4$
$x = 3$

The solution is (3,1). The solution set is $\{(3,1)\}$.

35. Since the length, y, is 5 times the width, x, then $y = 5x$.

38. Since each auto travels 3 hours, then A travels $3x$ miles and B travels $3y$ miles. They are 500 miles apart after 3 hours, so $3x + 3y = 500$.

Review exercises

1. $x = \frac{1}{3}$ **2.** $2x - 3y = -20$ **3.** $4x - 7y = -35$
4. $4(2x + y)(2x - y)$ **5.** $(3x - 2)^2$ **6.** $(5y + 4)(y - 2)$
7. $\{1\}$

Exercise 8–3

Answers to odd-numbered problems

1. $\{(3,-1)\}$ **3.** $\{(-1,3)\}$ **5.** $\{(-5,-1)\}$ **7.** $\{(5,2)\}$
9. $\left\{\left(-\frac{5}{3},-\frac{17}{3}\right)\right\}$ **11.** $\{(4,5)\}$ **13.** $\{(5,8)\}$ **15.** $\left\{\left(\frac{7}{5},\frac{1}{5}\right)\right\}$
17. $\left\{\left(\frac{2}{13},-\frac{14}{13}\right)\right\}$ **19.** no solution (inconsistent)
21. dependent (unlimited solutions) **23.** $\{(-1,0)\}$
25. $\left\{\left(-\frac{11}{13},-\frac{17}{13}\right)\right\}$ **27.** $\left\{\left(\frac{17}{20},\frac{1}{10}\right)\right\}$ **29.** $\{(4,-2)\}$
31. dependent (unlimited solutions) **33.** $\left\{\left(\frac{13}{4},\frac{7}{4}\right)\right\}$
35. inconsistent (no solution) **37.** $\left\{\left(5,\frac{21}{5}\right)\right\}$ **39.** $\left\{\left(\frac{28}{3},2\right)\right\}$
41. $\left\{\left(-\frac{7}{3},-\frac{26}{3}\right)\right\}$ **43.** $\left\{\left(0,\frac{1}{2}\right)\right\}$ **45.** $\{(3,-4)\}$
47. $\left\{\left(\frac{12}{13},-\frac{33}{13}\right)\right\}$ **49.** $\left\{\left(-\frac{76}{13},\frac{60}{13}\right)\right\}$ **51.** $\left\{\left(\frac{20}{17},-\frac{56}{17}\right)\right\}$
53. $x = 2y$ **55.** $x + y = 80$

Solutions to trial exercise problems

3. $-2x + 5y = 17$
$y = 2x + 5$
Substitute $2x + 5$ for y in the first equation.
$-2x + 5(2x + 5) = 17$
$-2x + 10x + 25 = 17$
$8x + 25 = 17$
$8x = -8$
$x = -1$
Substitute -1 for x in $y = 2x + 5$.
$y = 2(-1) + 5$
$y = -2 + 5$
$y = 3$
The solution is $(-1,3)$.
The solution set is $\{(-1,3)\}$.

17. $5x - 3y = 4$
$x + 2y = -2$ $\qquad x = -2y - 2$
Substitute $-2y - 2$ for x in $5x - 3y = 4$.
$5(-2y - 2) - 3y = 4$
$-10y - 10 - 3y = 4$
$-13y - 10 = 4$
$-13y = 14$
$$y = -\frac{14}{13}$$
Substitute $-\frac{14}{13}$ for y in $x = -2y - 2$.

Then $x = -2\left(\frac{-14}{13}\right) - 2 = \frac{28}{13} - \frac{26}{13} = \frac{2}{13}$

The solution is $\left(\frac{2}{13},\frac{-14}{13}\right)$.

The solution set is $\left\{\left(\frac{2}{13},-\frac{14}{13}\right)\right\}$.

35. $-3x - 2y = 6$
$6x + 4y = 5$
Solving for y, $-3x - 2y = 6$. (Multiply by -1.)
$3x + 2y = -6$
$2y = -6 - 3x$
$$y = -3 - \frac{3}{2}x$$
Substitute $-\frac{3}{2}x - 3$ for y in $6x + 4y = 5$.
$$6x + 4\left(-\frac{3}{2}x - 3\right) = 5$$
$6x - 6x - 12 = 5$
$-12 = 5$ (false)
The system is *inconsistent* so there is no solution.

42. $y = 3x - 4$
$y = 3x + 5$
Then $3x - 4 = 3x + 5$
$-4 = 5$ (false)
The system is *inconsistent* so there is no solution.

45. $x - 3 = 0$
$3x + 2y = 1$
Solving $x - 3 = 0$ for x, we have $x = 3$.
Substitute 3 for x in $3x + 2y = 1$.
$3(3) + 2y = 1$
$9 + 2y = 1$
$2y = -8$
$y = -4$
The solution is $(3, -4)$.
The solution set is $\{(3, -4)\}$.

48. $\frac{2x}{3} - \frac{y}{2} = 1$ Multiply by 6. $4x - 3y = 6$

$3x + \frac{y}{4} = 2$ Multiply by 4. $12x + y = 8$

Now $4x - 3y = 6$ $\quad$ $4x - 3y = 6$
$12x + y = 8$ Multiply by 3. $36x + 3y = 24$

$40x = 30$

$x = \frac{3}{4}$

Substitute $\frac{3}{4}$ for x in the equation $3x + \frac{y}{4} = 2$.

Then $3\left(\frac{3}{4}\right) + \frac{y}{4} = 2$

$\frac{9}{4} + \frac{y}{4} = 2$ Multiply by 4.

$9 + y = 8$
$y = -1$

The solution is $\left(\frac{3}{4}, -1\right)$.

The solution set is $\left\{\left(\frac{3}{4}, -1\right)\right\}$.

53. The first current, x, is twice as much as the second current, y, so $x = 2y$.

Review exercises

1. 16 ft and 26 ft **2.** 4 yd and 9 yd **3.** $b = \frac{2A - hc}{h}$ **4.** $\frac{2}{9}$

5. $\left\{\frac{5}{8}\right\}$ **6.** $-1 < x \leq 1$

Exercise 8–4

Answers to odd-numbered problems

1. $w = 4$ meters; $\ell = 14$ meters **3.** $w = 14$ meters; $\ell = 36$ meters **5.** $w = 11$ feet; $\ell = 20$ feet **7.** \$14,000 at 9%; \$4,000 at 7½% **9.** \$22,000 at 7%; \$8,000 at 9% **11.** \$13,300 at 9%; \$7,700 at 13% **13.** 4 feet and 8 feet **15.** 28 amperes and 52 amperes **17.** 38 teeth and 26 teeth **19.** 47 volts; 79 volts **21.** 19 ohms; 5 ohms **23.** 28 suits at \$125; 12 suits at \$185 **25.** 16 volts; 11 volts **27.** 1st gear—6 teeth; 2nd gear—17 teeth **29.** 80 cubic centimeters of 4%; 40 cubic centimeters of 10% **31.** 9 ounces of 80%; 3 ounces of 60% **33.** 87.5% **35.** $\frac{600}{7}$ centiliters of 3%; $\frac{40}{7}$ centiliters of 38% **37.** 25 mph; 75 mph **39.** Jane, $2\frac{1}{2}$ mph; Jim, 3 mph **41.** wind, 69 mph; airplane in still air, 391 mph **43.** $m = 3, b = -4; y = 3x - 4$ **45.** $m = -2, b = -1;$ $y = -2x - 1$ **47.** $m = \frac{4}{5}, b = -4$

Solutions to trial exercise problems

1. The perimeter P of a rectangle, by formula, is found by $P = 2\ell + 2w$, where ℓ is the length and w is the width of the rectangle. From "the perimeter of a rectangle is 36 meters," we get $2\ell + 2w = 36$. From "the length is 2 meters more than three times the width," we get $\ell = 3w + 2$.
We solve the system $2\ell + 2w = 36$
$\ell = 3w + 2$
Using substitution, substitute $3w + 2$ for ℓ in the first equation.
$2(3w + 2) + 2w = 36$ $\quad$ Then $\ell = 3w + 2$
$6w + 4 + 2w = 36$ $\quad$ $\ell = 3(4) + 2$
$8w + 4 = 36$ $\quad$ $\ell = 12 + 2$
$8w = 32$ $\quad$ $\ell = 14$
$w = 4$
The solution of the system is $(\ell, w) = (14, 4)$. The rectangle has length $\ell = 14$ m and width $w = 4$ m.

6. Let $x =$ the amount invested at 8% interest and $y =$ the amount invested at 6% interest. From "Phil has \$20,000, part of which he invests at 8% interest and the rest at 6%," we get $x + y = 20{,}000$. From "if his total income from the two investments was \$1,460," we get $0.08x + 0.06y = 1{,}460$. Multiply by 100 to clear decimal points: $8x + 6y = 146{,}000$.
We solve the system $x + y = 20{,}000$
$8x + 6y = 146{,}000$
Multiply the first equation by -6. $(-6)x + (-6)y = -6(20{,}000)$
$-6x - 6y = -120{,}000$
Add. $8x + 6y = 146{,}000$
$2x = 26{,}000$
$x = 13{,}000$
Substitute 13,000 for x in $x + y = 20{,}000$:
$13{,}000 + y = 20{,}000$, $y = 7{,}000$. Phil invested \$13,000 at 8% and \$7,000 at 6% interest.
Check: $(0.08)13{,}000 = \$1{,}040.00$
$(0.06)7{,}000 = \$\ \ 420.00$
Add to get income. $= \$1{,}460.00$

12. Let $x =$ the length of the longer piece of pipe and $y =$ the length of the shorter piece of pipe. From "a piece of pipe is 19 feet long," we get $x + y = 19$. From "one piece is 5 feet longer than the other piece," we get $x = y + 5$.
Solve the system: $x + y = 19$
$x = y + 5$
Use substitution: Substitute $y + 5$ for x in $x + y = 19$.
$(y + 5) + y = 19$
$2y + 5 = 19$
$2y = 14$
$y = 7$
Substitute 7 for y in $x + y = 19$.
$x + 7 = 19$
$x = 12$
The pipe is cut into pieces 12 feet and 7 feet long.

27. Let x = the number of teeth on the first gear and y = the number of teeth on the second gear. From "three times the number of teeth on a first gear is 1 more than the number of teeth on a second gear," we get $3x = y + 1$. From "five times the number of teeth on the first gear is 4 less than twice the number of teeth on the second gear," we get $5x = 2y - 4$.

Then $3x = y + 1$
$5x = 2y - 4$ is the system.

Rewrite the system.

$3x - y = 1$ Multiply by -2. → $-6x + 2y = -2$
$5x - 2y = -4$ $\quad 5x - 2y = -4$
Add: $-x = -6$
$x = 6$

Substitute 6 for x in $3x - y = 1$. $3(6) = y + 1$
$18 = y + 1$
$17 = y$

The first gear has 6 teeth and the second gear has 17 teeth.

29. Let x = the number of cubic centimeters (cc) of the 10% solution and y = the number of cc of the 4% solution.

Then $x + y = 120$ cc and $(0.10)x + (0.04)y = (0.06)120$.

Multiply the second equation by 100: $10x + 4y = 6(120)$, so $10x + 4y = 720$. Solve the system.

$x + y = 120$ Multiply by -4. $-4x - 4y = -480$
$10x + 4y = 720$ $\quad 10x + 4y = 720$
Add. $6x = 240$
$x = 40$

Substitute 40 for x in $x + y = 120$.
$40 + y = 120$
$y = 80$

Therefore the mechanic must have 40 cc of 10% solution and 80 cc of 4% solution.

37. Let x = the speed of the faster car and y = the speed of the slower car. Then using "if they drive toward each other they will meet in 1 hour," $t = 1$ for each car, so $x + y = 100$. Using "if they drive in the same direction they will meet in 2 hours," $t = 2$ for each car and the faster car travels 100 miles farther, so $2x - 2y = 100$.

We now have the system $x + y = 100$
$2x - 2y = 100$

Multiply the first equation by 2.
$2x + 2y = 200$
$2x - 2y = 100$
$4x = 300$
$x = 75$

Then since $x + y = 100$, $75 + y = 100$, $y = 25$. The average speeds are 75 mph and 25 mph.

40. Let x = the speed of the boat in still water and y = the speed of the current. Using $d = rt$ and (a) "a boat can travel 24 miles downstream in 2 hours," we get $2x + 2y = 24$; using $d = rt$ and (b) "and 16 miles upstream in the same length of time," we get $2x - 2y = 16$.

We have the system of equations $2x + 2y = 24$
$2x - 2y = 16$
$4x = 40$
$x = 10$

Then substituting 10 for x, $2(10) + 2y = 24$
$20 + 2y = 24$
$2y = 4$
$y = 2$

Therefore the boat travels at 10 mph in still water and the current is traveling at 2 mph.

43. Using $y = mx + b$, substitute
(a) 1 for x and -1 for y $\quad -1 = (1)m + b$
(b) 2 for x and 2 for y $\quad 2 = 2m + b$

Then $m + b = -1$ Multiply by -1. $-m - b = 1$
$2m + b = 2$ $\quad 2m + b = 2$
$m = 3$

Substitute 3 for m in $m + b = -1$.
$3 + b = -1$
$b = -4$

An equation of the line is $y = 3x - 4$.

Review exercises

1. $16x^4y^6$ **2.** $\dfrac{y}{x^3}$ **3.** $\dfrac{7}{12}$ **4.** $x^2 - 5x + 4$
5. $12x^4 + 8x^3 - 12x^2$ **6.** $9y^2 - 4$ **7.** $16x^2 - 40xy + 25y^2$
8. $\{-1\}$ **9.** $\{-2,5\}$

Exercise 8–5

Answers to odd-numbered problems

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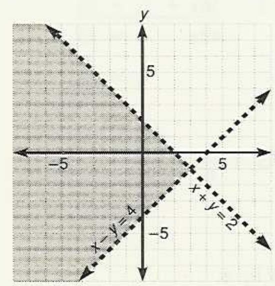

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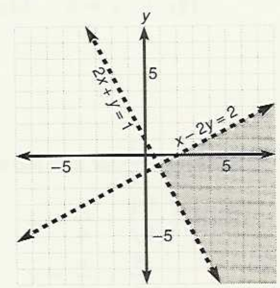

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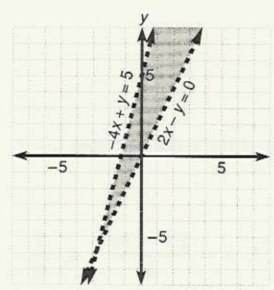

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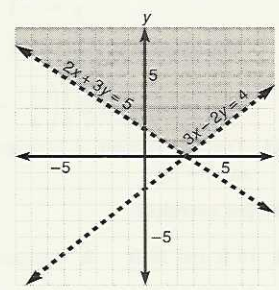

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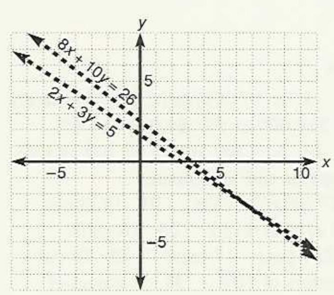

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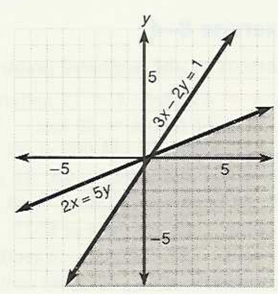

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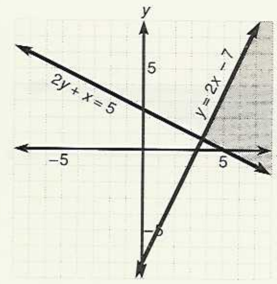

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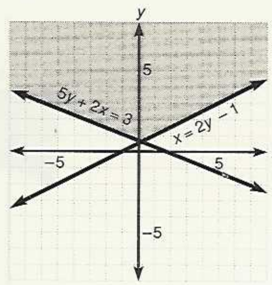

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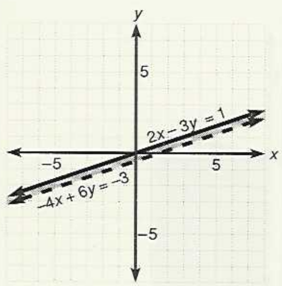

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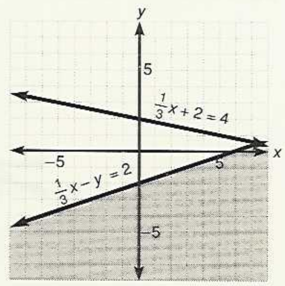

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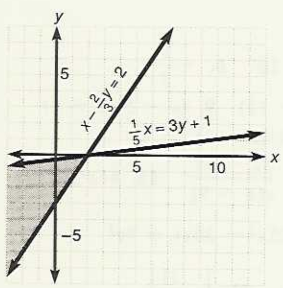

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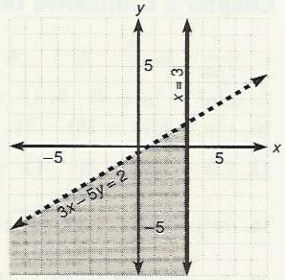

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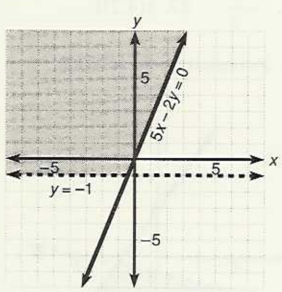

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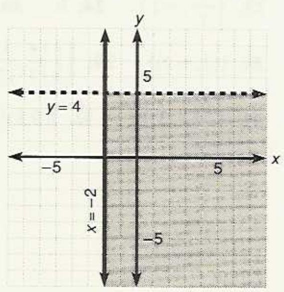

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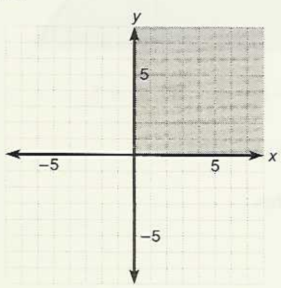
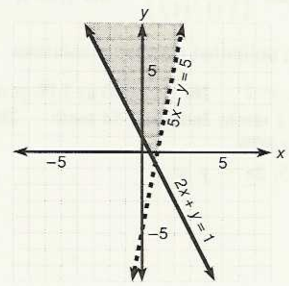


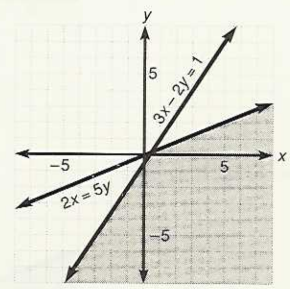
31. **a.** $x < 0$ $y < 0$ **b.** $x > 0$ $y < 0$

Solutions to trial exercise problems

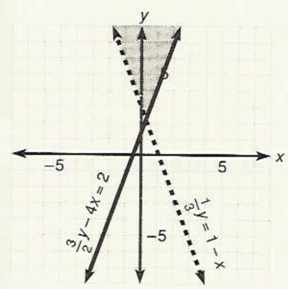
6. $5x - y < 5$
$2x + y \geq 1$



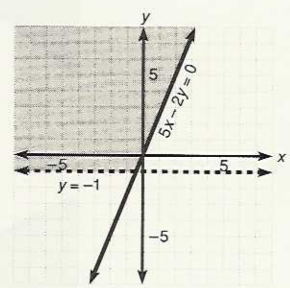
11. $3x - 2y \geq 1$
$2x \geq 5y$



22. $\frac{3}{2}y - 4x \geq 2$
$\frac{1}{3}y > 1 - x$
(*Note:* Clear fractions.)
$3y - 8x \geq 4$
$y > 3 - 3x$



25. $5x - 2y \leq 0$
$y > -1$



Review exercises

1. $2x^2 + 3x + 7$ **2.** $4x^2 - 11x - 3$ **3.** $25y^2 - 20y + 4$ **4.** $49z^2 - 1$ **5.** a^4 **6.** xy^2 **7.** $-y^9$ **8.** $x(5x - 3)$ **9.** $(2x - 1)(2x + 5)$ **10.** $(3y - 5)^2$

Chapter 8 review

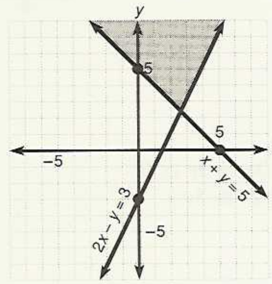
1. $\{(3,1)\}$ **2.** $\{(2,4)\}$ **3.** $\{(4,2)\}$ **4.** $\{(-2,6)\}$ **5.** inconsistent; no solution **6.** dependent; unlimited number of solutions **7.** $\{(3,0)\}$ **8.** $\{(7,2)\}$ **9.** $\{(-1,1)\}$ **10.** $\{(1,-1)\}$

11. $\{(0,2)\}$ **12.** $\{(5,-3)\}$ **13.** $\{(5,2)\}$ **14.** inconsistent; no solution **15.** $\{(2,1)\}$ **16.** dependent; unlimited number of solutions **17.** $\{(5,3)\}$ **18.** $\{(5,-3)\}$ **19.** inconsistent; no solution **20.** $\left\{\left(\frac{24}{11},\frac{5}{11}\right)\right\}$ **21.** $\{(3,1)\}$ **22.** $\{(7,2)\}$

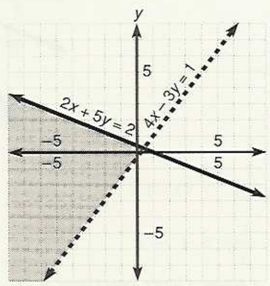
23. dependent; unlimited number of solutions **24.** $\left\{\left(\frac{19}{11},-\frac{17}{11}\right)\right\}$

25. $\ell = 25$; $w = 7$ **26.** \$11,000 at 14% profit; \$7,000 at 23% loss **27.** 1st gear, 12 teeth; 2nd gear, 54 teeth **28.** 30 milliliters at 60%; 90 milliliters at 80%

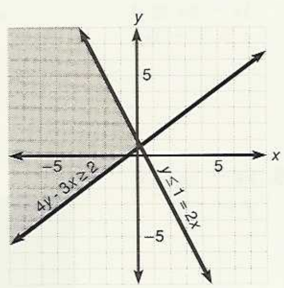
29. $x + y \geq 5,\ 2x - y < 3$



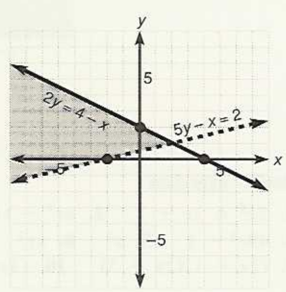
30. $4x - 3y < 1,\ 2x + 5y \leq 2$



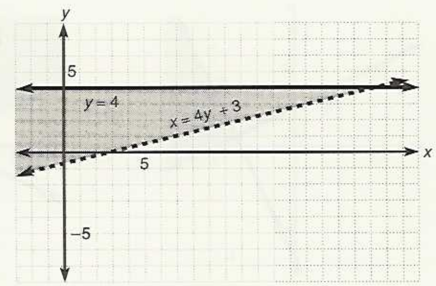
31. $4y - 3x \geq 2,\ y \leq 1 - 2x$



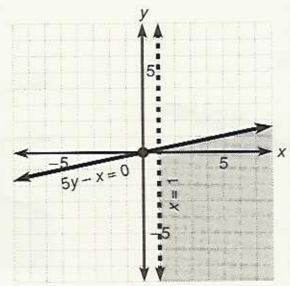
32. $2y \leq 4 - x,\ 5y - x > 2$



33. $x > 4y + 3,\ y \leq 4$

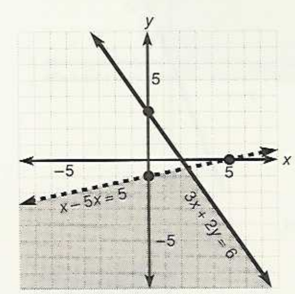


34. $x > 1,\ 5y - x \leq 0$



Chapter 8 cumulative test

1. $-\frac{1}{64}$ **2.** $\frac{4}{3}$ **3.** 1 **4.** $\frac{1}{25}$ **5.** 16 **6.** $-\frac{1}{3}$
7. $3a(a^2 + 5a + 9)$ **8.** $(x + 11)(x - 11)$ **9.** $(a - 5)(x + y)$
10. $(3a + b)(a - 2b)$ **11.** $7x(x + 1)(x - 1)$
12. $(5b - 2)(b - 4)$ **13.** $\frac{y^2}{2x}$ **14.** $\frac{32 - 3x}{x(x - 9)}$ **15.** $81x^4y^6z^6$
16. $9x^2y - 14y^2$ **17.** $\frac{a - 7}{7}$ **18.** $25x^2 - 30xy + 9y^2$
19. $2y + 6$ **20.** $\left\{\frac{15}{8}\right\}$ **21.** $\{0,-9\}$ **22.** $\left\{-\frac{11}{36}\right\}$
23. $\left\{\frac{3}{7},-1\right\}$ **24.** 6 **25.** $\frac{4}{5}$ **26.** $-\frac{1}{3}$ **27.** $\{(4,2)\}$
28. $\{(1,-2)\}$ **29.** $\left\{\left(\frac{14}{17},\frac{1}{17}\right)\right\}$ **30.** inconsistent; no solution
31. $\left\{\left(\frac{5}{8},1\right)\right\}$ **32.** $\{(0,8)\}$ **33.**



34. 6, 19 **35.** 42, 37 **36.** 5, 6 or −6, −5 **37.** 0 or 5

38. 1 or 2 **39.** $\frac{20}{9}$ hours or $2\frac{2}{9}$ hours

40. length = 13 inches; width = 7 inches

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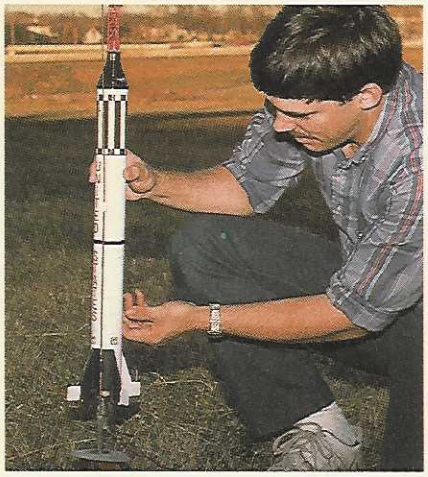

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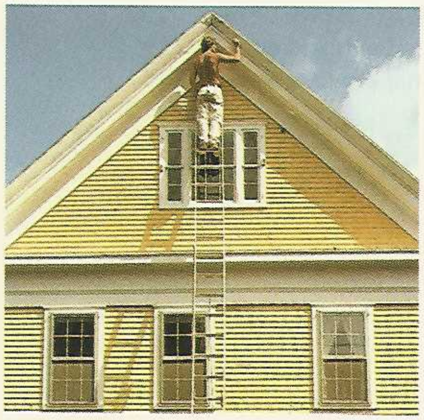

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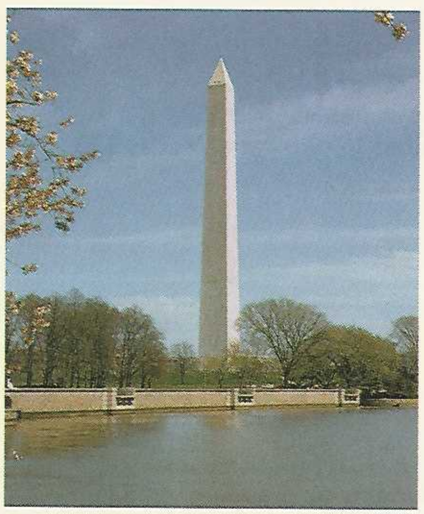

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